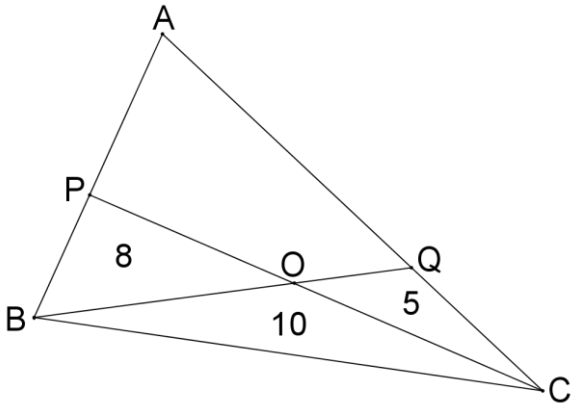
# Areas and Ratios

Can you rearrange the statements below to find the area of the quadrilateral AQOP?

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|---|---|
| A | Substituting this back into equation (1) gives $E = 12$. |
| B | Triangles $QAO$ and $OAB$ have the same height above the line $BQ$, so $\frac{F}{OQ} = \frac{E+8}{OQ}$. |
| C | Triangles $PAO$ and $OAC$ have the same height above the line $PC$, so $\frac{E}{PO} = \frac{F+5}{CO}$. |
| D | Combining these two equations gives $\frac{E}{8} = \frac{F+5}{10}$. |
| E | The area of quadrilateral $AQOP$ is $E + F = 10 + 12 = 22$. |
| F | Triangles $PBO$ and $OBC$ also have the same height above the line $PC$, so $\frac{8}{PO} = \frac{10}{CO}$. |
| G | Solving for $F$ gives $F = 10$. |
| H | Doubling equation (2) and rearranging gives $10E = 20F - 80$. |
| I | Draw in the line $AO$. Label the area of triangle $AOP$ as $E$, and that of $AOQ$ as $F$. |
| J | Clearing denominators gives $10E = 8F + 40$. Call this equation (1). |
| K | Clearing denominators gives $10F = 5E + 40$. Call this equation (2). |
| L | Combining these two equations gives $\frac{F}{5} = \frac{E+8}{10}$. |
| M | Triangles $QCO$ and $OBC$ also have the same height above $BQ$, so $\frac{5}{OQ} = \frac{10}{BO}$. |
| N | Substituting this into equation (1) gives $20F - 80 = 8F + 40$. |