

Imagine a machine with two lights. When you enter a number, a light turns on if your number satisfies one of the machine's rules. If your number satisfies both rules, both lights will switch on.

Here are some examples of possible sequence rules that might be generated by the machine:

$$\begin{array}{ccc} 8n + 3 & 4n + 1 & 11n + 10 \\ 6n + 5 & 3n + 2 & 5n + 1 \\ 3n + 1 & & \\ 12n + 4 & 10n + 1 & 9n + 3 \end{array}$$

Identify some pairs of rules for which it is possible to turn both lights on simultaneously. What do you notice about their rules?

For each pair of rules, find a number that would turn both lights on. Can you find a way of generating a whole sequence of such numbers?

Identify some pairs of rules for which it is **not** possible to turn both lights on simultaneously. What can you say about their rules that convinces you that it is not possible?

EXTENSION:

If the two sequences are described by the rules $an+b$ and $cn+d$, can you explain the conditions for determining whether the lights will ever switch on together?