x^2+3x+4

(A) $\left[\frac{1}{7}, 7\right]$ (B) $\left[\frac{1}{3}, 3\right]$ (C) $\{1\}$ (D) None of These.

Solution: Let $f(x) = y$

$$\Rightarrow \frac{x^2-3x+4}{x^2+3x+4} = y$$

$$\Rightarrow x^2-3x+4 = x^2y+3xy+4y$$

$$\Rightarrow 0 = x^2y - x^2 + 3xy + 3x + 4y - 4$$

$$\Rightarrow x^2(y-1) + 3x(y+1) + 4(y-1) = 0$$

This equation ~~of~~ is of the form $ax^2+bx+c=0$

where $a = y-1$

$b = 3(y+1)$

and $c = 4(y-1)$

$\therefore$ Discriminant $(D) = b^2 - 4ac$

$$= [3(y+1)]^2 - 4(y-1)4(y-1)$$

$$= 9(y^2+2y+1) - 16(y^2-2y+1)$$

$$= 9y^2+18y+9-16y^2+32y-16$$

$$= -7y^2+50y-7$$

$$= -(7y^2-50y+7)$$

i.e. $D < 0$

$$\Rightarrow -(7y^2-50y+7) < 0$$

$$\Rightarrow 7y^2-50y+7 \geq 0$$

$\Rightarrow y \in \left[\frac{1}{7}, 7\right]$

$\cancel{\Rightarrow f(x) \in \left[\frac{1}{7}, 7\right]}$

$\Rightarrow \text{Range}\,(f(x)) \in \left[\frac{1}{7}, 7\right]$

Hence option (A) is correct.