Jiali Huang NRICH problem.

Curvy equation

First part

Sketch the graph of function h

$$h(x) = \frac{\ln x}{x} \quad (x>0)$$

X intercept would be found by equating the equation to 0

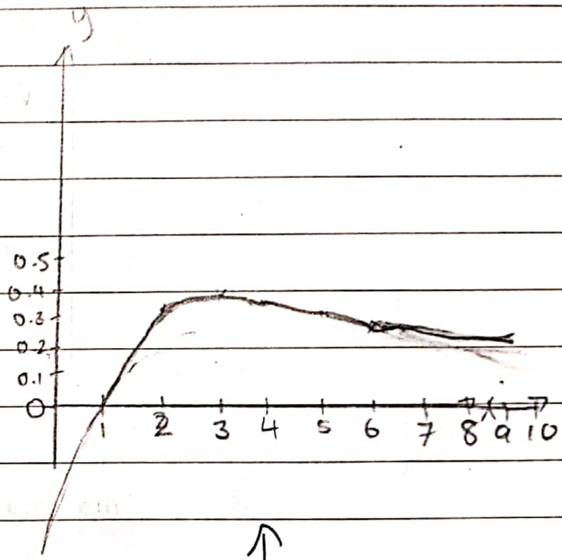
$$\frac{\ln x}{x} = 0$$

$$\ln x = 0$$

$$\log_e x = 0$$

$$e^0 = 1$$

$$x = 1$$



substituting values of x close to zero

$x = 0.01$ $\quad$ $x = 0.5$

$\frac{\ln(0.01)}{0.01}$ $\quad$ $\frac{\ln(0.5)}{0.5}$

-460.5 $\quad$ -1.4

shows there's a ~~an~~ vertical asymptote at $x=0$

Substituted $x=2$ $\quad x=3$ $x=4$ etc... to find the y values

$\frac{\ln(2)}{2} = 0.347$ $\quad$ $\frac{\ln(3)}{3} = 0.366$ $\quad$ $\frac{\ln(4)}{4} = 0.347$

$\frac{\ln(5)}{5} = 0.321$ $\quad$ $\frac{\ln(6)}{6} = 0.299$ $\quad$ $\frac{\ln(7)}{7} = 0.28$

the y values decrease as x increases ~~at the~~. at ~~around~~ afterwards.

Second part

all pairs of distinct positive integers which satisfy the equation.

$$n^m = m^n$$

taking the natural log of both.

$$\ln n^m = \ln m^n$$

$$m \ln(n) = n \ln(m)$$

✓ ~~the 2 solutions~~

$$\frac{\ln(n)}{n} = \frac{\ln(m)}{n}$$

by inspection

$2^4 = 4^2$

$n = 2$

$m = 4$

This is the only solution and can be proven when linking to the first part of the question.

by differentiating

$$y = \frac{\ln x}{x}$$

Using the quotient rule

$$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{x\frac{1}{x} - \ln(x)\times 1}{x^2}$$

$$= \frac{1 - \ln(x)}{x^2}$$

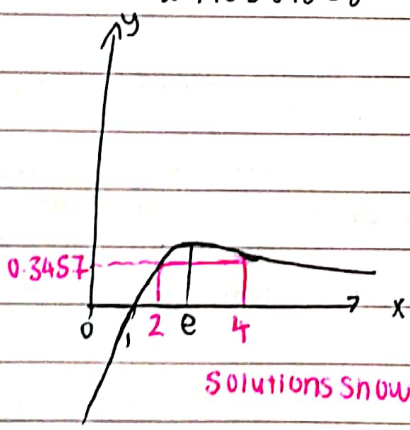
maximum point

$$\frac{1-\ln(x)}{x^2} = 0$$

$$\ln(x) = 1$$

$$\begin{bmatrix} x = e \\ y = 0.3678794412 \end{bmatrix}$$

$e = 2.718281828$



e is the maximum point, so the only integer values that could work below e are 1, 2

Solutions shown on the graph, both left and right to e

$4\ln 2 = 2\ln 4$

$$\frac{\ln(2)}{2} = \frac{\ln(4)}{4} =$$

$= 0.34657$

$2 < e < 4$

$~~2^4 > 4^2~~$

log Equivalence

$4\ln 2 = 2\ln 4$

$\frac{1}{2}\ln 2 = \frac{1}{4}\ln 4$

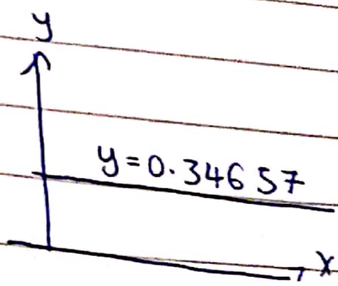
$= 0.3465735903$

represents the equivalent y value on the curve at points $x=2$ and $x=4$

$$\frac{(\log n)}{n} = \frac{(\log m)}{n}$$

$n\log m = m\log n$

$\longrightarrow$ would give the graph



only 2 and 4 satisfy this when $n<e$ and $m>e$

greatest y value is when

$$y = \frac{\ln(e)}{e} = 0.368$$

graph of $\frac{\ln 2}{2}$ and $\frac{\ln 4}{4}$