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 I first started looking for patterns which were universal and had no exceptions. They were:

$$0, y = 0 = x, 0$$

$$1, y = y$$

$$x, 1 = x$$

I then started looking for patterns in each row. I found one rule, but it had several exceptions.

$$x, y = x + y + 1$$

I started looking at the exceptions and found another universal rule with no exceptions.

If x is a multiple of y, then y would equal the purple number.  
 Or  
 If y is multiple of x, then x would equal the purple number.

I still had exceptions and I started collecting them and tried to look for a pattern in them. After looking for a pattern excessively, I found this rule which I thought at the time, worked for almost everything except for my first four rules which I discovered.

$$\frac{n-1}{n}x + y$$

Or

To get n, make x/y and then simplify to n/m. Now we know the value of n.

$$\frac{n-1}{n}y + x$$

To get n, make y/x and then simplify to n/m. Now we know the value of n.

Then after a little more analysis, I discovered that this rule also applies to the second, third and fourth rule. Though, the new rule still didn't cover my first rule.

After analyzing my rules with my teacher. We were able to simplify my rule so that it would cover all cases and had no exceptions. We realized that n was x or y divided by the Highest Common Factor which we called z and substituted it into my rule.

$$\frac{\frac{x}{z} - 1}{\frac{x}{z}}x + y$$

We then started simplifying:

$$\frac{\frac{x}{z} - \frac{z}{z}}{\frac{x}{z}}x + y$$

$$\frac{x-z}{\frac{x}{z}}x + y$$

We turned one into z/z so we could do the operation

We then did:

$$\frac{\frac{x-z}{z}}{\frac{x}{z}} = \frac{x-z}{z} \cdot \frac{z}{x} = \frac{x-z}{x}$$

These two cancel out

And substituted it into my rule

$$\frac{x-z}{x} x + y$$

And These two operations cancel each other out and we get:

$$x - z + y$$

Which is equal to:

$$x + y - z$$

And so we discovered the rule which applies to every coordinate and has no exceptions is the value of the x axis and the y axis combined minus their highest common factor.