An Appearing Act: Stage 3 and 4 - Toughnut — Zach Thompson

The first thing that springs to mind is that 5, 8 and 13 are consecutive Fibonacci numbers. I wondered if the pattern can be repeated on a larger and/or smaller scale.

1
1
2
3
5 → $5 \times 13 = \mathbf{65}$
8 → $8^2 = 64$
13
21
34
55 → $55 \times 144 = 7920$
89 → $89^2 = \mathbf{7921}$
144
233
377
610
987

This how the question appears in the Fibonacci sequence. Here the rectangle is bigger.

This time the rectangle is smaller than the square, but the difference is still equal to one.

I decided to try out some other "triplets", so see if I could find a pattern. My process was to: take 3 consecutive Fibonacci numbers, square the middle number and multiply the first and third. This is what I found:

Fibonacci set	Square the middle number	Multiple the first and third numbers	Which is bigger?		Observations:
			Square	Rectangle	
{1,1,2}	$1^2 = 1$	$1 \times 2 = \mathbf{2}$		✓	This is the smallest set.
{1,2,3}	$2^2 = \mathbf{4}$	$1 \times 3 = 3$	✓		There is a constant difference of one. This alternates between the square and the rectangle (starting with the rectangle as the biggest).
{2,3,5}	$3^2 = 9$	$2 \times 5 = \mathbf{10}$		✓	
{3,5,8}	$5^2 = \mathbf{25}$	$3 \times 8 = 24$	✓		
{5,8,13}	$8^2 = 64$	$5 \times 13 = \mathbf{65}$		✓	
{8,13,21}	$13^2 = \mathbf{169}$	$8 \times 21 = 168$	✓		
{13,21,34}	$21^2 = 441$	$13 \times 34 = \mathbf{442}$		✓	
{21,34,55}	$34^2 = \mathbf{1156}$	$21 \times 55 = 1155$	✓		
{34,55,89}	$55^2 = 3025$	$34 \times 89 = \mathbf{3026}$		✓	
{55,89,144}	$89^2 = \mathbf{7921}$	$55 \times 144 = 7920$	✓		

If the rectangle is bigger, there will be a gap of one unit^2, but if the square is bigger then there will be an overlap of one unit^2 in its associated rectangle.

I thought I should try some bigger numbers, just to check:

			Square	Rectangle
{1597,2584,4181}	$2584^2 = 6677056$	$1597 \times 4181 = \mathbf{6677057}$		✓
{2584,4181,6765}	$4181^2 = \mathbf{17480761}$	$2584 \times 6765 = 17480760$	✓	

I did go on and try even bigger numbers, and the pattern continued to hold.

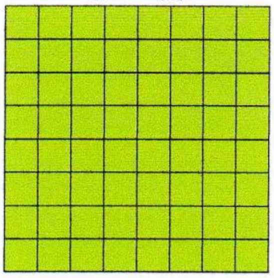
This seems to be an interesting feature of Fibonacci numbers, but it's only because I noticed the sequence, that I was able to explore this relationship!

To answer the final question: the smaller the set, the more obvious the overlap or gap. The larger the numbers get, the harder it is to see with the naked eye. If you use coloured squares and count them, it is much clearer:

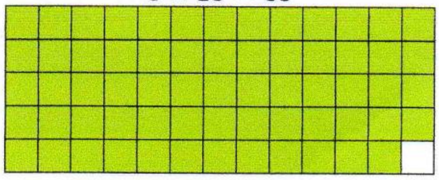
a) $2^2 = 4$ $1 \times 3 = 3$

Overlap of 1unit^2

b) $8^2 = 64$



$5 \times 13 = 65$



Gap of 1unit^2

The overlap in the set (a) is equal to the gap in set (b), but because the diagonal is smaller in (a) than (b), this same difference is spread over a smaller area, so it should be a little more obvious.

I used quite small squared paper, so it was difficult to spot the gap in (b) when I chopped up the square to make a rectangle, even though I knew it must be there. If I had used much larger graph paper, the gap would have been a little more noticeable. I guess it depends upon the scale of your diagram. If the scale is the same, it's the very smallest sets that should be most noticeable.

I love Fibonacci numbers!!!!