Wallington High School for Girls

Enrich - Wipeout

Section 1.1 **Investigation:**

* Sequence no. -

1. 1, 2
2. 1, 2, 3, 4
3. 1, 2, 3, 4, 5, 6
4. 1, 2, 3, 4, 5, 6, 7, 8
5. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
6. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

These are the first 6 sequences, where N (the last number) is even

A Table[1] to Show what the Mean would be for each Sequence, if I wipeout each number

THE NUMBER WIPED OUT

THE SEQUENCE NO.	1	2	3	4	5	6	7	8	9	10	11	12
1	2	1	-	-	-	-	-	-	-	-	-	-
2	3	$\frac{8}{3}$	$\frac{7}{3}$	2	-	-	-	-	-	-	-	-
3	4	$\frac{19}{5}$	$\frac{18}{5}$	$\frac{17}{5}$	$\frac{16}{5}$	3	-	-	-	-	-	-
4	5	$\frac{34}{7}$	$\frac{33}{7}$	$\frac{32}{7}$	$\frac{31}{7}$	$\frac{30}{7}$	$\frac{29}{7}$	4	-	-	-	-
5	6	$\frac{53}{9}$	$\frac{52}{9}$	$\frac{51}{9}$	$\frac{50}{9}$	$\frac{49}{9}$	$\frac{48}{9}$	$\frac{47}{9}$	$\frac{46}{9}$	5	-	-
6	7	$\frac{76}{11}$	$\frac{75}{11}$	$\frac{74}{11}$	$\frac{73}{11}$	$\frac{72}{11}$	$\frac{71}{11}$	$\frac{70}{11}$	$\frac{69}{11}$	$\frac{68}{11}$	$\frac{67}{11}$	6

KEY:

- = the number isn't in the sequence

○ = the mean that is a whole number

Section 1.2 **Common Patterns found in this Table:**

1. When the last or first number is wiped out from each Sequence respectively, this is the only case where the mean is a whole number. For example, when the number '1' from the Sequence '⑤' is wiped out, you get a 6 (which is a whole number). Also, when you wipe out '10 (the last number)' from this same sequence, you get a 5 (another whole number). Hence, when N (the last number of this consecutive number sequence) is even, you can only wipeout the first or last number (N)

Section 1.4 Problem 1 and 2

Question: Which numbers can be wiped out, so that the mean of what is left is a whole number? Can you explain why? What happens when N is odd?

Explained previously (using Table [1]), we have found out that to obtain a whole number mean, once a number is wiped out, you can only wipe out the first / last number (only if the last number is even). This section will explore further on this point using different methods and investigations:

Method 1 - Using diagrams, pictures and facts

Before we start, we must consider one important fact that will help us solve this problem:

- In a consecutive number sequence, the mean is always the median (only if N (the last number) is **odd**)

To look deeper into this fact we will follow a similar investigation as what we did when 'N = Even', but this time, the condition will change to 'N = Odd':

* Sequence no. -

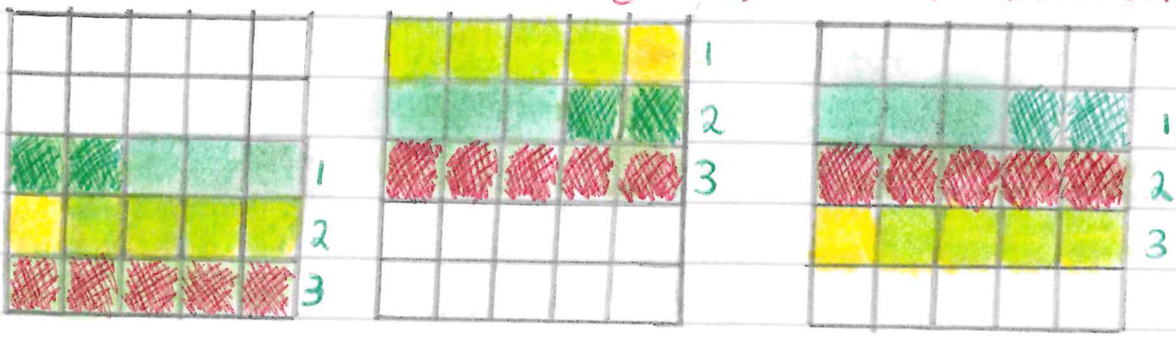
7 1, 2, 3
8 1, 2, 3, 4, 5
9 1, 2, 3, 4, 5, 6, 7
10 1, 2, 3, 4, 5, 6, 7, 8, 9
11 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

These are the first 6 sequences, where 'N = odd'

A Two-Way Table [2] to show the correlation between the mean and the median of each sequence:

	7	8	9	10	11
Mean	2	3	4	5	6
Median	2	3	4	5	6

This shows us that the mean is the same as the median, with the condition 'N = odd' in one row and another block of the same row to a different row as the first block. Below, you will see 3 examples once rearranged, but if you remember that **all the blocks in a single row must move together and must add up to the highest / last number in a consecutive number sequence**, then there are more than 3 ways (but they still all lead to the same answer): FOLLOW THE COLOUR CODED DIAGRAM OF SEQUENCE NO. 8 FOR THIS REARRANGEMENT



With the help of colour coding each row, we have discovered that there are 3 complete rows, hence the mean is 3. Therefore, we can deduce that the mean is the median (proven with this diagram that mean and median was 3). One important thing to remember is that this DIAGRAM can only work if N is odd and the starting number is odd; also, it must be a series (consecutive number sequence). → However, the 'summary rule' shown below can work with any series sequence, as long as there is an odd amount of values

Summary:

In a series sequence from 1 to N (where N is odd), the mean is the median.

ection 1.5 * Now that we have proved this theory, we will now find what number needs to be wiped out (from the 1 to N (N = odd) sequence) to make the remaining mean a whole number:

We will be following the same sequence numbers from the previous investigation, where N = odd

A Table[3] to show what the mean would be for each sequence, if I wipe out each respective no.

has an odd amount of values. However, we know that the 1 to N (N=even) sequence has an even amount of values. We cannot add another value to make the values odd, but we can <u>wipe out</u> a value to make an odd amount of values. One important thing to remember is that this only works in a <u>series sequence</u>, which means a consecutive number sequence or a sequence with the intervals between each value being the same. This means that you cannot wipe out any middle value, as it disturbs the intervals, hence, you can only remove the last or first (no.) value, if you want the mean / median to be a whole number.

2. Algebraic calculations : → only works if 'N = even'

- If you remove the largest value, N, the remaining mean is '$\frac{N}{2}$'
- If you remove the smallest value, 1, the remaining mean is '$\frac{N}{2} + 1$'

→ All the other possible means will lie inbetween these 2 consecutive means and as all the values are whole numbers, these 2 consecutive means are also whole numbers. We know that all the values between 2 consecutive whole numbers will be decimals, hence if you need whole number mean, you can only wipe out 1 or N.

ANS

Section 3

Problem 1 Answer:

Deduced from the following investigations, if N is even (following the 1 to N Sequence :

- You need to wipe out the first or last value to get a whole number for the remaining mean

PLEASE READ 'Section 2' FOR THE REASONING AND EXPLANATION OF THIS THEORY