Abhinav Tavva NRICH Mega Quadratic Equations 5/7/22

$$(x^2-5x+5)^{(x^2-11x+30)} = 1$$

For an exponential equation to be equal to 1, either:

1. The index is 0 and the base is any number (apart from 0).
2. The base is 1
3. The base is -1 and the index is an even number

To solve this mega quadratic equation, we solve 3 quadratics:

1. $x^2-11x+30=0$
$(x-6)(x-5)=0$
$x=6$ or $x=5$

2. $x^2-5x+5=1$
$x^2-5x+4=0$
$(x-4)(x-1)=0$
$x=4$ or $x=1$

3. $x^2-5x+5=-1$
$x^2-5x+6=0$
$(x-3)(x-2)=0$
$x=3$ or $x=2$ (The index value of these solutions are even).

The six solutions are: 1, 2, 3, 4, 5, 6

① $(x^2-7x+11)^{(x^2-13x+42)}$

Using the same method:

1. $x^2-13x+42=0$
$(x-7)(x-6)=0$
$x=7$ or $x=6$

2. $x^2-7x+11=1$
$x^2-7x+10=0$
$(x-5)(x-2)=0$
$x=5$ or $x=2$

8. $x^2-7x+11=-1$
$x^2-7x+12=0$
$(x-4)(x-3)=0$
$x=4$ or $x=3$ (The index values of these solutions are even)

The six solutions are: 2, 3, 4, 5, 6, 7

These solutions are all 1 higher than previous

Using the 2 examples, we can form 3 rules:

- First, let the 6 solutions be A, B, C, D, E, F

1. The index of the mega quadratic is $(x-E)(x-F)$
2. The base of the equation is $(x-A)(x-D)+1$
3. The base of the equation can also be $(x-B)(x-C)-1$

② Six Solutions: 3, 4, 5, 6, 7, 8

1. The index value is $(x-7)(x-8) = x^2 - 15x + 56$
2. The base value is $(x-3)(x-6)+1 = x^2 - 9x + 19$
3. The base value is $(x-4)(x-5)-1 = x^2 - 9x + 19$

Therefore, the mega quadratic is $(x^2-9x+19)^{(x^2-15x+56)}$

Another six Solutions: 4, 5, 6, 7, 8, 9

1. Index $= (x-8)(x-9) = x^2 - 17x + 72$
2. Base $= (x-4)(x-7)+1 \,/\, (x-5)(x-6)-1 = x^2 - 11x + 29$

Final answer $= (x^2 - 11x + 29)^{(x^2-17x+72)}$

③ $(x^2-5x+5)^{(x^2-4)} = 1$

1. $x^2 - 4 = 0$
   $(x+2)(x-2) = 0$
   $x = 2$ or $x = -2$

2. $x^2 - 5x + 5 = 1$
   $x^2 - 5x + 4 = 0$
   $(x-4)(x-1) = 0$
   $x = 4$ or $x = 1$

3. $x^2 - 5x + 5 = -1$
   $x^2 - 5x + 6 = 0$
   $(x-3)(x-2) = 0$
   $x = 3, x = 2$

There is repetition of the solution 2, and the solution of 3 in the 3rd Step does not create an even index when it is Substituted into the index equation, as $3^2 - 4 = 5$, which is an odd number. $(-1)^5 = -1$ not 1, hence two of the solutions can be removed.

This leaves us with these 4 solutions: $-2, 1, 2, 4$

④ $(x^2-6x+10)^{(x^2+x-2)} = 1$

1. $x^2+x-2=0$
$(x-1)(x+2)=0$
$x=1$ or $x=-2$

2. $x^2-6x+10=1$
$x^2-6x+9=0$
$(x-3)(x-3)$
$x=3$

3. $x^2-6x+10=-1$
$x^2-6x+11=0$
$\downarrow$
The discriminant (b^2-4ac) of this quadratic is negative, because, $(-6)^2-4\times1\times11=-8$, hence there are no real solutions for when the base is -1.

This leaves us with the three solutions; $-2, 1, 3$

⑤ For a mega quadratic with only 2 solutions, I would need to use the same idea as in question 4 where the discriminant of a quadratic is negative, so I will re-use the base of $x^2-6x+10$ to remove any values for -1 and to only provide 1 solution for when the base is equal to 1.

For the index value, I can use any quadratic with a repeated root that isn't 3, such as x^2-4x+4

Using these 2 quadratics, we can form a reasonable equation:

$$(x^2-6x+10)^{(x^2-4x+4)}$$

The two solutions are: 2, 3

For a mega quadratic with 5 solutions, we can again re-use the idea in a previous question. This time we can use ~~the repeated root idea for the index, such as $x^2+10x+25$~~ the difference of two squares idea, such as x^2-25
For the base, we can use a quadratic with a solution to when it is equal to -1 is an odd number for the index, such as x^2-5x+5 (the solution of 2 is forms an index of -21)

We can therefore form the mega quadratic equation of:

$$(x^2-5x+5)^{(x^2-25)}$$

The five solutions are: $-5, 1, 3, 4, 5$