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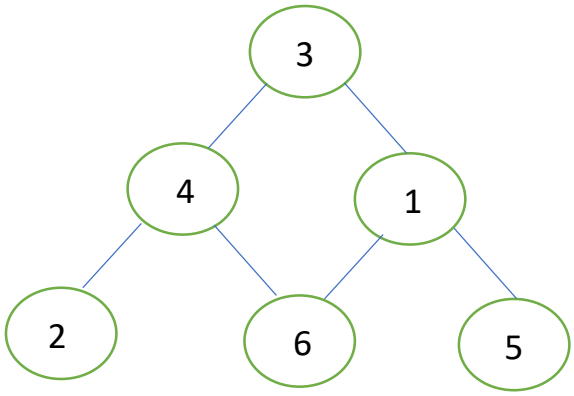
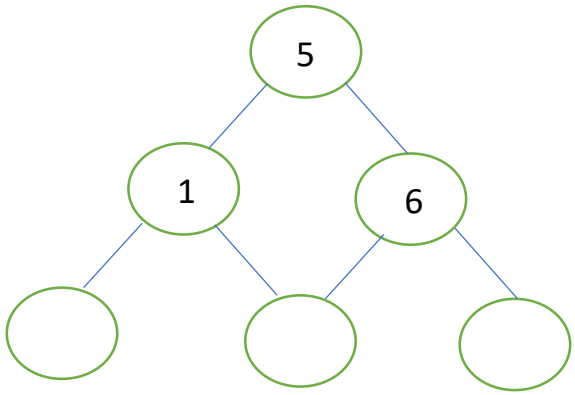
6 must be on the bottom row?

6 cannot be made from the difference of the other numbers from 1 to 5. Since this is impossible then it must always be placed on the bottom row as shown below:



5 cannot be at the top?

5 can only be made from the difference between 6 and 1. However, in this instance, this would place 6 on the middle row as shown below:



We already know that 6 can only be placed on the bottom row, therefore it is also impossible for 5 to be at the top.

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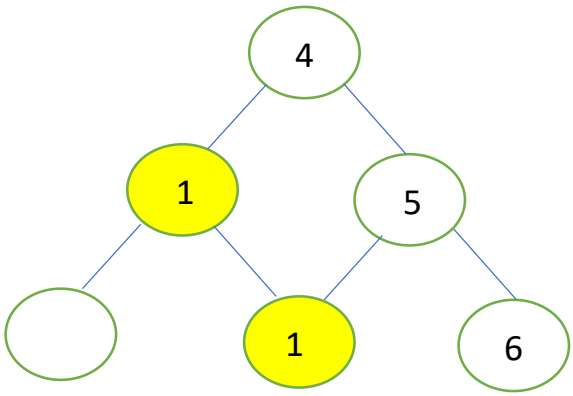
4 cannot be at the top?

4 can be made from the difference between 6 and 2 and 5 and 1. However, we already know that 6 can't be placed on the middle row. Therefore, the pairing of 6 and 2 cannot be used in the middle row below 4. We can now consider the pairing of 5 and 1 in the middle row:



We already know that 5 can only be made from the difference between 6 and 1 and we know that 6 would work on the bottom row. However, the above diagram shows that the value 1 (highlighted in yellow) has already been used in the middle row and so we are unable to use the 6 and 1 pairing on the bottom row. Therefore, it is impossible for 4 to be at the top.

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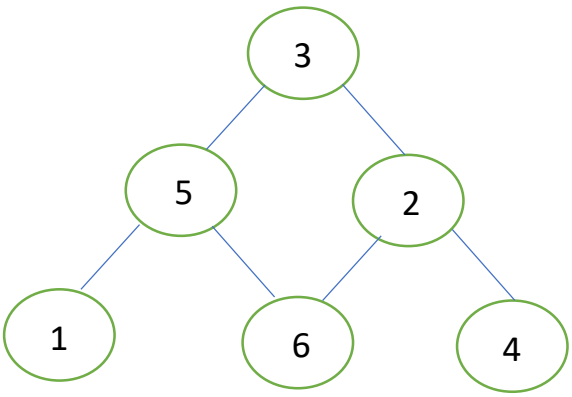
All possible solutions

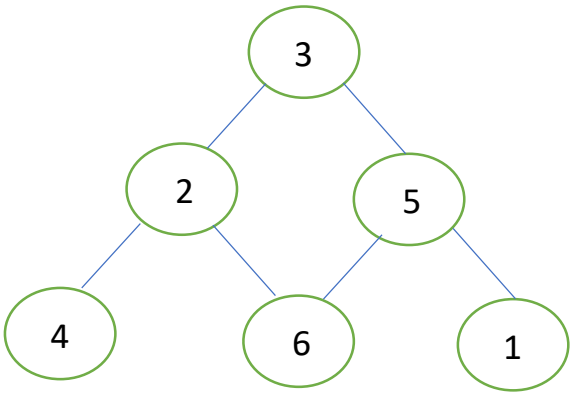
The investigation so far has proved that values 6, 5 and 4 cannot go on the top. Therefore, I tested placing 3, 2 and 1 at the top to get all the possible solutions.

a) 3 at the top

Possible pairings on the middle row would be 6 and 3, 5 and 2 and 4 and 1. We already know that 6 can only be on the bottom row but also the 3 would be a duplicate value. Therefore, the pairing of 6 and 3 on the middle row is impossible.

If we now consider the pairing of 5 and 2 on the middle row, there are two possible solutions:





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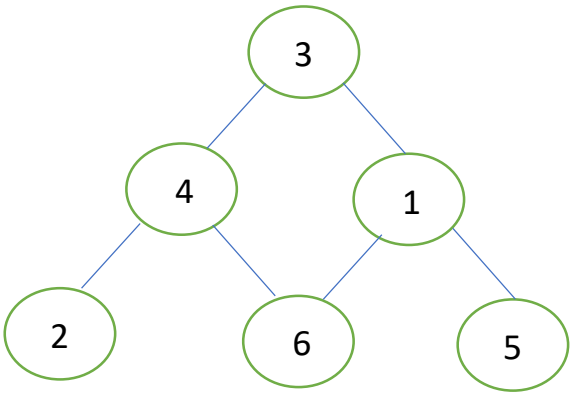
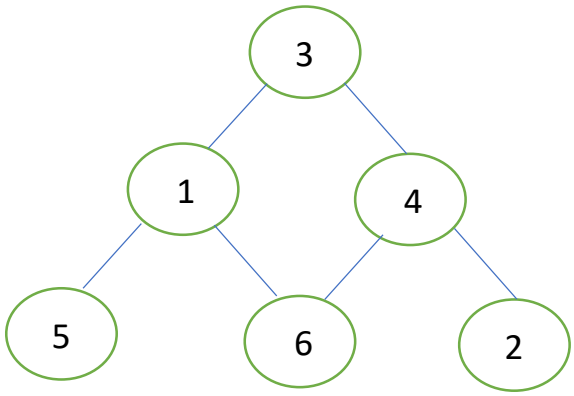
a) 3 at the top (continued)

If we now consider the pairing of 4 and 1 on the middle row, there are another two possible solutions:

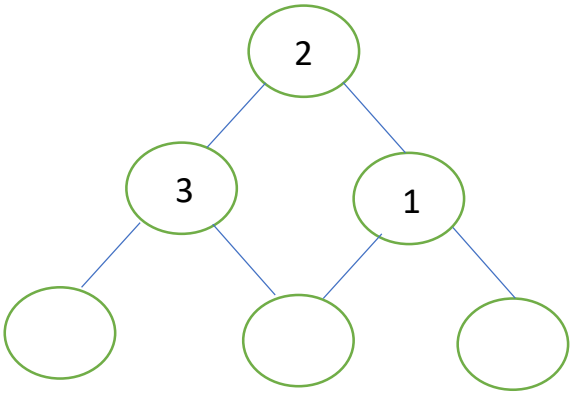




b) 2 at the top

Possible pairings on the middle row would be 6 and 4, 5 and 3, 4 and 2 and 3 and 1. We already know that 6 can only be on the bottom row but also the 2 would be a duplicate value. Therefore, the pairings of 6 and 4 and 4 and 2 on the middle row are impossible.

If we now consider the pairing of 3 and 1 on the middle row:



This pairing would leave the values 4, 5 and 6 to be placed on the bottom row. However, it would be impossible to place these in any combination that would actually work. Therefore, the pairing of 3 and 1 in the middle row is impossible.

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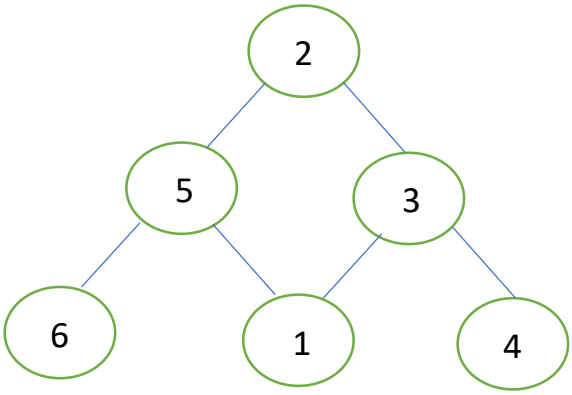
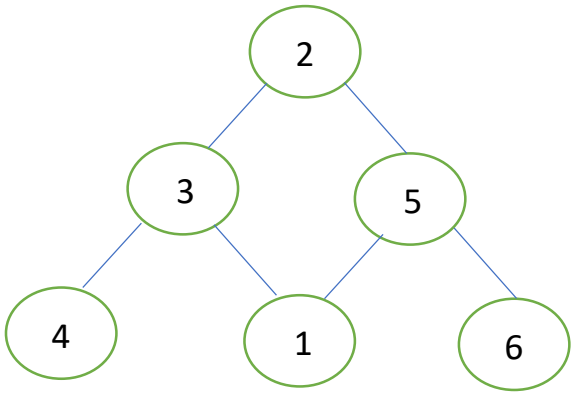
b) 2 at the top (continued)

If we now consider the pairing of 5 and 3 on the middle row, there are also two possible solutions:

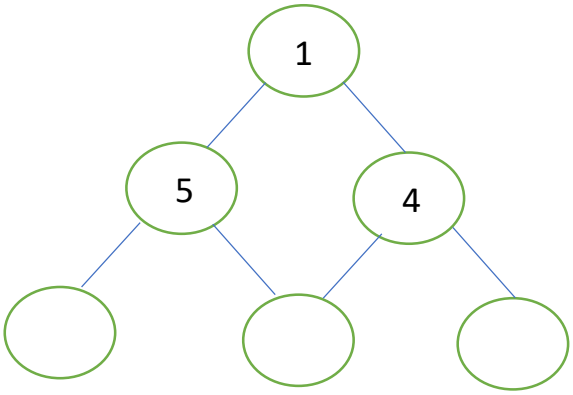


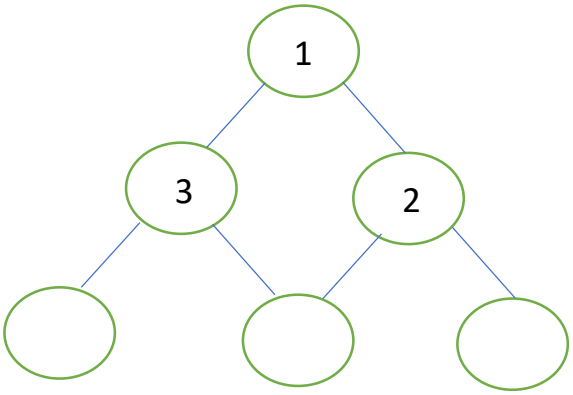


c) 1 at the top

Possible pairings on the middle row would be 6 and 5, 5 and 4, 4 and 3 and 3 and 2. We already know that 6 can only be on the bottom row so the pairing of 6 and 5 can be excluded.

If we now consider the pairings of 5 and 4 and 3 and 2 on the middle row. In both instances we would be unable to place the remaining values on the bottom row:





It is impossible to place the remaining values of 6,3,2.

It is impossible to place the remaining values of 6,5,4.

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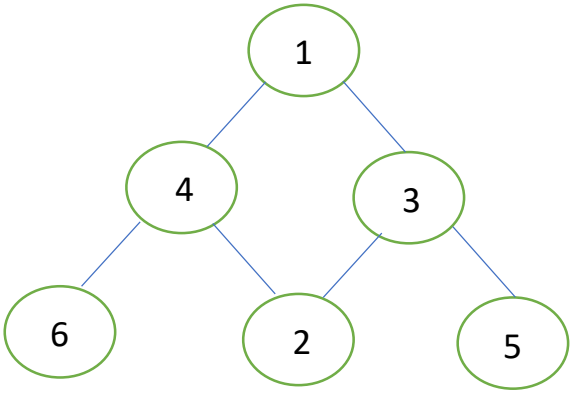
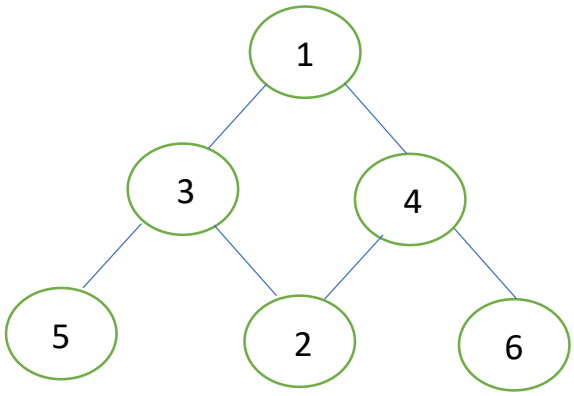
c) 1 at the top (continued)

If we now consider the pairing of 4 and 3 on the middle row, there are also two possible solutions:

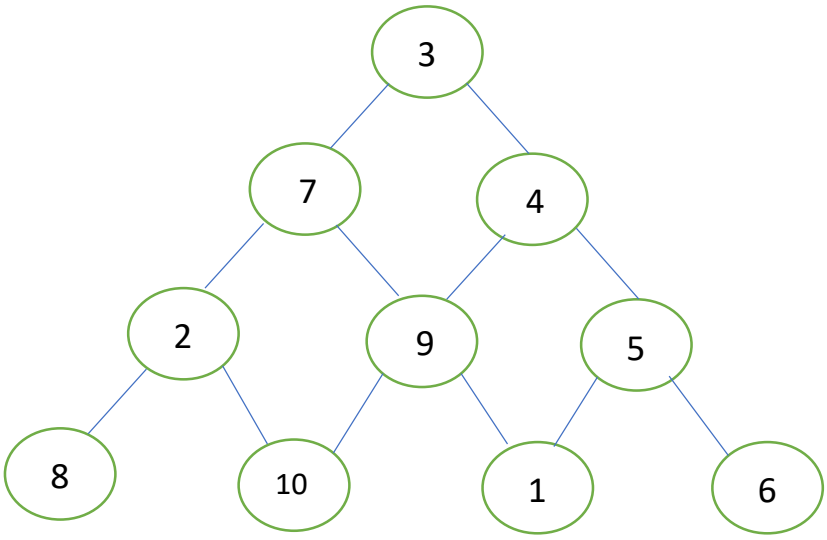




Therefore, to fill the circles with numbers from 1 to 6 there are 8 possible solutions.

Extension:

Here is a solution for the numbers 1 to 10:



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Another solution can be obtained by swapping the values in the 2nd row: