Pythagoras Perimeters - Sophie Walker

Proving Area = 36 - c

First, we know: $a+b+c=12$, $a^2+b^2=c^2$, $Area = \frac{ab}{2}$

Next to have a and b on one side in all equations;

$a+b+c=12 \Rightarrow a+b=12-c$

Squaring both sides gives

$(a+b)^2 = (12-c)^2$

$(a+b)(a+b) = (12-c)(12-c)$

$a^2+2ab+b^2 = 144-24c+c^2$

According to Pythagoras theorem $a^2+b^2=c^2$, therefore we can remove c^2 for both sides.

$a^2+2ab+b^2 = 144-24c+c^2$

$-(a^2+b^2) \quad 2ab = 144-24c \quad -c^2)$

Now to get the area we need to divide by 4 so $2ab$ becomes $\frac{ab}{2}$ which is the area

$\div 4 \quad 2ab = 144-24c \quad \div 4$

$\frac{ab}{2} = 36-6c$

$\therefore Area = 36-6c$

Proving Area = 225-15c

This time, we can use the same method as before, despite $a+b+c=30$

$a+b=30-c$

$(a+b)^2=(30-c)^2$

$-(a^2+b^2) \quad a^2+2ab+b^2 = 900-60c+c^2 \quad -c^2$

$\div 4 \quad 2ab = 900-60c \quad \div 4$

$\frac{ab}{2} = 225-15c \qquad \therefore Area = 225-15c$

Finding a General Equation for Area (with hypotenuse and perimeter)

To find this equation, the perimeter will be P instead of a number and the hypotenuse is c. Therefore;

$a+b+c=P$, $a^2+b^2=c^2$, $\text{Area} = \frac{ab}{2}$

$-c \quad a+b+c=P \quad -c$

$a+b=P-c$

$(a+b)^2=(P-c)^2$

$-(a^2+b^2) \quad a^2+2ab+b^2=P^2-2Pc+c^2 \quad -c^2$

$\div 4 \quad 2ab=P^2-2Pc \quad \div 4$

$\frac{ab}{2} = \frac{P^2-2Pc}{4}$

$\frac{ab}{2} = \frac{1}{4}P(P-2c)$ $\therefore \text{Area} = \frac{1}{4}P(P-2c)$

To further prove this, we can substitute P with 12 or 30.

$P=12$

$\text{Area} = \frac{1}{4}P(P-2c)$

$= \frac{1}{4} \times 12 \times (12-2c)$

$= 3 \times (12-2c)$

$= 36-6c$

$P=30$

$\text{Area} = \frac{1}{4}P(P-2c)$

$= \frac{1}{4} \times 30 \times (30-2c)$

$= 7\frac{1}{2} \times (30-2c)$

$= 225-15c$

From the Previous work we know these areas are correct, $\therefore$ proving:

$\text{Area} = \frac{1}{4}P(P-2c)$