DATE: 21 Oct. 2015

Starter

I started with a two-digit number, then divided it by 2 and multiplied the answer by 9, before reversing the digits. The answer was the same as the original number. What number did I choose?

Let the two-digit number = $10a + b$

Since the answer was the same as the original number,

the answer = $10a + b$.

When you divide the starting two-digit number by 2 and multiply by 9, it becomes:

$10a + b$ [÷2]

$\frac{10}{2}a + \frac{1}{2}b$

$5a + \frac{1}{2}b$ [×9]

$45a + \frac{9}{2}b$

When you turn the answer to the stage before reversing the digits, it becomes:

$10b + a$

Since the stage before reversing is the stage after multiplying by 9,

$45a + \frac{9}{2}b = 10b + a$

If you simplify the equation, it becomes:

$45a + \frac{9}{2}b = 10b + a$ [×2]

$90a + 9b = 20b + 2a$

$88a = 11b$

$8a = b$

Since $0 < a < 10$ and $0 \leq b < 10$, and a, b = positive integers

the only possible pair for a and b is 1 and 8.

So, $a = 1$, $b = 8$.

Therefore, the original number is 18.

· I take another two-digit number, add 1 then divide by 2. If I reverse the digits I get the original number. What number did I start with?

Let the original two-digit number $10a+b$.
When you add 1, it becomes $10a+b+1$.
Since the last number you get after reversing is also $10a+b$,
the number before reversing, it's $10b+a$.
When you multiply $10b+a$ by 2, to go one step backward, befor dividing by 2,
it becomes: $20b+2a$.
Since,
$10a+b+1 = 20b+2a$
$8a+1 = 19b$

$8a$ is always even.
So, $8a+1$ is always odd.
So, b should be always odd.
If b is even $19b$ becomes even so $8a+1=19b$ doesn't make sense.

When $a=9$, $8a+1=73$.
$19\times3=57$, $19\times4=76$.

So, the biggest possible b is 3.

$b=0, 1, 2, 3$
$19b=0, 19, 38, 57$.

among 0, 19, 38 and 57,
the only number with a form of $8a+1$ is 57.

So $8a+1=57$
$8a=56$
$a=7$
$57=19\times3$
$b=3$.

Therefore, the original number is 73.

· Choose a two digit number, subtract 10, divide by 2 and reverse the digits. What number should you choose to get your original number?

Let the original number $=10a+b$.
the final number is also $10a+b$

before reversing and dividing by 2, the number is $= 20b+2a$
after subtracting 10, the number is $= 10a+1-10$

Since, $20b+2a=10a+b-10$,

$19b=8a-10$
$8a=19b+10$

When $a=9$,

$9a = 72$.

$72 - 10 = 62$.

$19 \times 3 = 57$, $19 \times 4 = 76$.

So, the biggest possible b is 3.

$b = 0, 1, 2, 3$

$9b + 10 = 10, 29, 48, 67$.

among those 4 numbers, the only number that is a multiple of 8 is 48.

$48 = 8 \times 6$.

Therefore, $a = 6$, $b = 2$, and the original number is 62.