Difference of two squares by Adithya Venkat (14)

Choose a number in the 3 times table.Take the numbers on either side of your chosen number and find the difference between their squares.
Try it a few times. What do you notice?
Can you prove it will always happen?

1) $4^2-2^2= 16-4 = 12$
2) $7^2-5^2= 49-25=24$
3) $10^2-8^2=100-64=36$

I clearly have noticed that the result of the difference of the squares of two numbers either side of a multiple of three results in a multiple of 12 for the numbers I have used above. However , is this pattern constant?

Let n= a multiple of 3,
$(n+1)^2-(n-1)^2= n^2+2n+1-(n^2-2n+1)$
$(n+1)^2-(n-1)^2= 4n$

Therefore for any two numbers either side of a multiple of three , denoted by n, the difference in their squares gives a result 4 times the number n or in other words 4 times the multiple of three, thus meaning the pattern is the same throughout.

Choose a number in the 5 times table.
Take the numbers on either side of your chosen number and find the difference between their squares.
Try it a few times. What do you notice?
Can you prove it will always happen?

1) $6^2-4^2=36-16=20$
2) $11^2-9^2=121-81=40$
3) $16^2-14^2=256-196=60$

The transparent pattern here once again is that the difference of the squares of two numbers either side of a multiple of 5 gives a multiple of 20 for the numbers used above. Does it always work?

Let n= a multiple of 5,
$(n+1)^2-(n-1)^2= n^2+2n+1-(n^2-2n+1)$
$(n+1)^2-(n-1)^2= 4n$

Thus , in this instance , the difference in the squares of any two numbers either side of a multiple of five, denoted by n, results in 4 times the number n or in other words 4 times the multiple of five, therefore the pattern is the same throughout.

Is there a similar relationship for other times tables?

Yes. There is definitely a similar relationship for numbers either sides of a multiple in all times tables. To show this pattern below are numbers adjacent to multiples of six,seven and eight.

Six:

1) $7^2-5^2=49-25=24$
2) $13^2-11^2=169-121=48$
3) $19^2-17^2=361-289=72$

Here the answers ascend in multiples of 24. When n= a multiple of 6,$(n+1)^2-(n-1)^2=n^2+2n+1-(n^2-2n+1)$, so $(n+1)^2-(n-1)^2= 4n$. As a result the end product is always 4 times the number n, denoting a multiple of 6 in this circumstance, and more specifically always a multiple of 24 also.

Seven:

1) $8^2-6^2=64-36=28$
2) $15^2-13^2=225-169=56$
3) $22^2-20^2=484-400=84$

For every multiple of seven, four times the number n always produces a multiple of 28.

Eight:

1) $9^2-7^2=81-49=32$
2) $17^2-15^2=289-225=64$
3) $25^2-23^2=625-529=96$

For all multiples of eight, the result which is 4 times the number n always produces a multiple of 32.

Instead of taking the numbers on either side of your starting number, investigate what happens if you take the numbers two above and two below your starting number and then work out the difference between their squares…

Let's take multiples of 4,5 and 6:

Four:

1) $6^2-2^2=36-4=32$
2) $10^2-6^2=100-36=64$
3) $14^2-10^2=196-100=96$

By taking numbers two away from a multiple of 4, the answer is always a multiple of 32. This is because when n= a multiple of 4, $(n+2)^2-(n-2)^2= n^2+4n+4-(n^2-4n+4)$, so $(n+2)^2-(n-2)^2= 8n$. Therefore any multiple of 4,represented by n, when multiplied by 8 will be the result of the difference of the squares of two numbers two away from a multiple of 4.

Five:

1) $7^2-3^2=49-9=40$

2) $12^2-8^2=144-64=80$
3) $17^2-13^2=289-169=120$

Taking into account that when n= a multiple of 5 here , $(n+2)^2-(n-2)^2= n^2+4n+4-(n^2-4n+4)$, so $(n+2)^2-(n-2)^2= 8n$, the end result is always 8 times the multiple of 5 and also a multiple of 20.

<u>Six:</u>

1) $8^2-4^2=64-16=48$
2) $14^2-10^2=196-100=96$
3) $20^2-16^2=400-256=144$

As I've previously establishes, these set of numbers which are two away from multiples of seven always produce an answer that is 8 times the multiple of n when the difference of their squares are calculated; here the answers are multiples of 48.

By increasing the gap between the multiples of a certain number, the end result is the multiple n multiplied by a multiple of 4 depending on the interval of the numbers relative to the number n . For example, interval one gives an answer of 4n as shown above whilst interval of two produces 8n. Interval of three unsurprisingly is in accord with the established pattern, producing 12n due to the fact that $(n+3)^2-(n-3)^2=n^2+6n+9-n^2+6n-9=12n$.