A general strategy could be to follow along with Alison's method which is accomplished by rearranging the soldiers into 4 congruent rectangles. For the case of 960 soldiers, we can do the same.

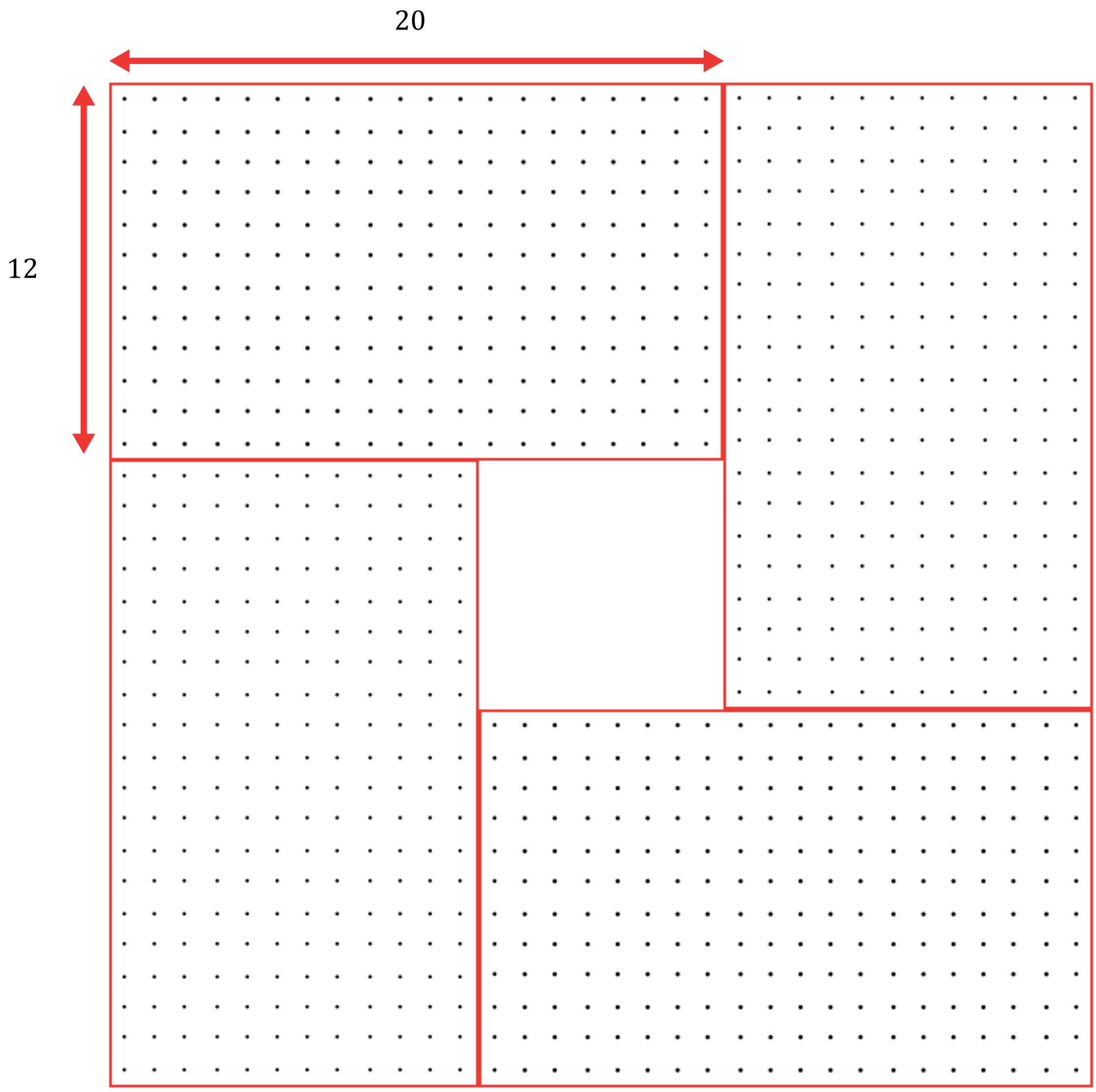
$$\frac{960}{4} = 240$$

Each rectangle will contain a group of 240 soldiers because all four groups of rectangles contain the same number of soldiers. Many various dimensions of rectangle for 240 soldiers can be formed. They are listed below

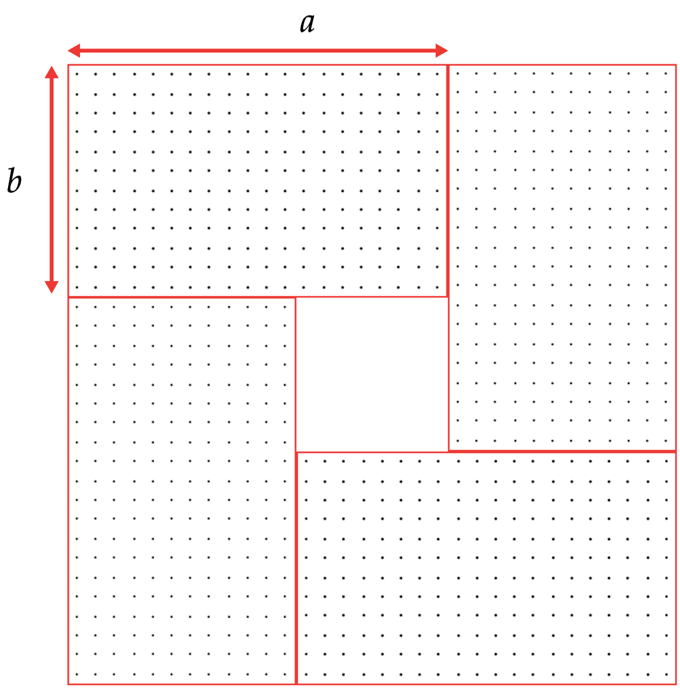
$$1 \times 240 = 240$$
$$2 \times 120 = 240$$
$$3 \times 80 = 240$$
$$4 \times 60 = 240$$
$$5 \times 48 = 240$$

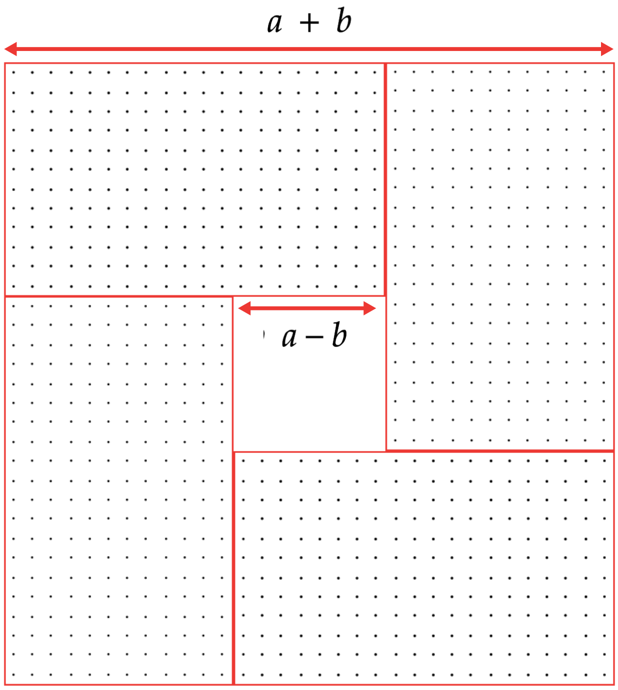
$$6 \times 40 = 240$$
$$8 \times 30 = 240$$
$$10 \times 24 = 240$$
$$12 \times 20 = 240$$
$$15 \times 16 = 240$$

Each such dimension with 4 copies of itself properly arranged can show that it indeed forms a hollow square. Below, one such dimensions is illustrated.



Let ‘a’ and ‘b’ be the two numbers when multiplied give 240 $a > b$ $a \times b = 240$





$$(a + b)^2 - (a - b)^2 = 960$$
$$(a^2 + b^2 + 2ab) - (a^2 + b^2 - 2ab) = 960$$
$$a^2 + b^2 + 2ab - a^2 - b^2 + 2ab = 960$$
$$4ab = 960$$
$$ab = 240$$

Substitute the factor pairs obtained for 240.

$$(241)^2 - (239)^2 = 960$$
$$(122)^2 - (118)^2 = 960$$
$$(83)^2 - (77)^2 = 960$$
$$(64)^2 - (56)^2 = 960$$
$$(53)^2 - (43)^2 = 960$$
$$(46)^2 - (34)^2 = 960$$
$$(38)^2 - (22)^2 = 960$$
$$(34)^2 - (14)^2 = 960$$
$$(32)^2 - (8)^2 = 960$$
$$(31)^2 - (1)^2 = 960$$

Thus there are total 10 ways in which the general can arrange his 960 soldiers

Let n be the number of soldiers, then to form a hollow square formation,

$$n = (a + b)^2 - (a - b)^2$$
$$n = \left(a^2 + b^2 + 2ab\right) - \left(a^2 + b^2 - 2ab\right)$$
$$n = a^2 + b^2 + 2ab - a^2 - b^2 + 2ab$$
$$n = 4ab$$

So, to form a hollow square formation the number of soldiers should always be a multiple of 4. Hence the battalion sizes that can't be arranged as symmetric hollow squares are never a multiple of 4

General Strategy

1. First take n the number of soldiers and divide it by 4. You get (n / 4) which must be an integer as shown above. Let (n / 4) = k
2. List out the factor pairs possible for k
3. For each pair acquired where k = ab (where a and b two positive integers & a > b),

$$n = (a + b)^2 - (a - b)^2$$

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