

# Hollow Squares:

A Symmetric 960?

- 1<sup>st</sup> Find total number of ways.
- 2<sup>nd</sup> How many are symmetrical?

① we are finding out a literal difference of two squares.

$$\therefore x^2 - y^2 = 960 \equiv (x+y)(x-y) = 960$$

req  $x, y$ : positive integers.

↳ Therefore, one can simply write out the factors.

	960	
1	<del>960</del>	960
2	482	480
3	<del>323</del>	320
4	244	240
5	<del>197</del>	192
6	166	160
8	128	120
10	106	96
12	92	80
15	<del>79</del>	64
16	76	60
20	68	48
24	64	40
30	62	32
The sum.		

We know it can only work when the two factors sum to an even number

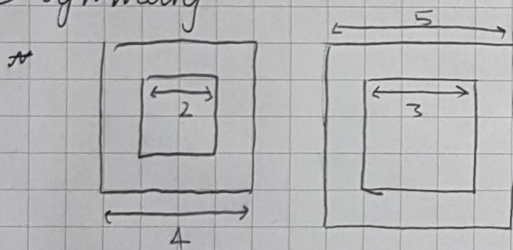
$$\therefore x+y+x-y = 2x = 2(x) \therefore \text{must be even.}$$

↳  $\therefore$  can add the factors and eliminate the odd sums.

↳ As one can see, there are 10 possible ways to get 960 by taking away a smaller square from a larger square

↳ However, not all are symmetrical!

## ② Symmetry



We can see that symmetry is created when both numbers are either odd or even

$$\begin{aligned} \text{odd} + \text{odd} &= \text{even} \\ \text{even} + \text{even} &= \text{even} \end{aligned}$$

• Aim: filter out the results for  $x$  and  $y$  (from  $x^2 - y^2 = 960$ ) in which  $x$  and  $y$  are not both even or odd.

X1) 2, 480  
 $x+y = 480$   
 $x-y = 2$

$$2x = 482$$

$$x = 241 \text{ (E)}$$

$$241 - y = 2$$

$$y = 239 \text{ (O)}$$

↳ not symmetrical.

↳ because  $x+y > x-y$  as  $y$  cannot be negative.

2) 4, 240  
 $2x = 244$   
 $x = 122 \text{ (E)}$

$$122 - y = 4$$

$$y = 118 \text{ (E)}$$

One can make the process simpler by comparing value of  $x$  and lowest factor.  
 ↳ if both even or both odd it works.

symmetrical.

$$\times 3) \textcircled{E} 6, 160$$

$$2x = 166$$

$$x = 83 \textcircled{O}$$

No symmetry.

$$\checkmark 4) \textcircled{E} 8, 120$$

$$2x = 128$$

$$x = 64 \textcircled{E}$$

Symmetry

$$\times 5) \textcircled{E} 10, 96$$

$$2x = 106$$

$$x = 53 \textcircled{O}$$

No symmetry

$$\checkmark 6) \textcircled{E} 12, 80$$

$$2x = 92$$

$$x = 46 \textcircled{E}$$

Symmetry.

$$\checkmark 7) \textcircled{E} 16, 60$$

$$2x = 76$$

$$x = 38 \textcircled{E}$$

Symmetry.

$$\checkmark 8) \textcircled{E} 20, 48$$

$$2x = 68$$

$$x = 34 \textcircled{E}$$

Symmetry

$$\checkmark 9) \textcircled{E} 24, 40$$

$$2x = 64$$

$$x = 32 \textcircled{E}$$

Symmetry

$$\times 10) \textcircled{E} 30, 32$$

$$2x = 62$$

$$x = 31 \textcircled{O}$$

No symmetry

Out of the 10 possible ways to form a 960 hollow square, only 6 ways give a symmetric hollow square.