



● Quadrature of The lunes 1

Area of lune = area of semicircle - area of segment

Area of Segment


Area of sector = $\frac{90}{360} \times \pi \times 2^2$
 $= \pi \text{ cm}^2$


Area of triangle = $\frac{1}{2} \times 2 \times 2 \times \sin(90)$
 $= 2 \text{ cm}^2$

Area of segment = sector - triangle
 $= \pi - 2$
 $= 1.141592653\dots$

● Because angle in semicircle = 90° , line AB is diameter of ~~the~~ circle which is composed of lune.

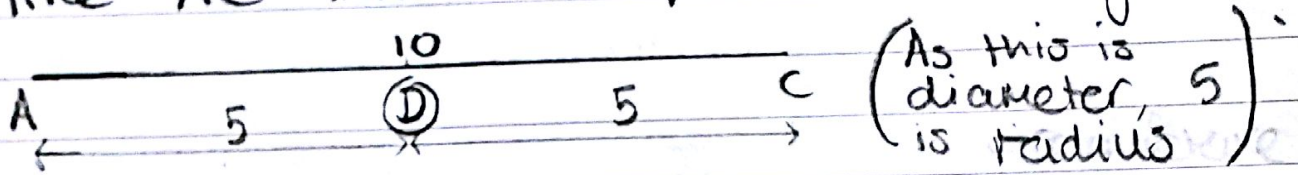
$$AB^2 = 2^2 + 2^2$$
$$AB = 8$$
$$AB = 2\sqrt{2}$$

Area of semicircle = $\frac{1}{2} (\pi \times \sqrt{2}^2)$
 $= \frac{1}{2} (\pi \times 2)$
 $= \pi$

Area of lune = $\pi - 1.141592653\dots$
 $= 2 \text{ cm}$

● Quadrature of the Lunes 3

First we need to mark on point D which splits line AC into two equal 5 unit lengths.



Then you can draw a line from point B to point D, we know that this line is 5 units because the line goes from B which is on the circumference of the white circle to D, the centre of the circle.

Angle \hat{BCA}

$$\cos x = \frac{8}{10}$$

$$x = \cos^{-1}\left(\frac{8}{10}\right)$$

$$x = 36.86\dots$$

From the rule:

the angle at the centre =

2 x the angle at circumference
(provided both angles are from the same chord)

we can say that

$$\begin{aligned}\text{angle } \hat{BDA} &= 2x \\ &= 73.73\dots\end{aligned}$$

From the angles on a straight line + to 180° rule, we can deduce that

$$\begin{aligned}\text{angle } \hat{BDC} &= 180 - 73.73\dots \\ &= 106.26\dots\end{aligned}$$

Now we have the needed angles we can work out the lune areas.

Blue lune

$$\text{Sector area} = \frac{73.73...}{360} \times \pi \times 5^2 = 16.08...$$

$$\text{Triangle area} = \frac{1}{2} \times 5 \times 5 \times \sin(73.73...) = 12$$

(using sine rule)

$$\text{Segment area} = 16.08... - 12 = 4.08...$$

Line AB is the diameter because angles in semicircle = 90° (Thales Theorem)

$$\text{Semicircle} = \frac{1}{2} \times \pi \times 3^2 = 14.13...$$

$$\text{Lune} = \text{semicircle area} - \text{segment area}$$

$$= 14.13... - 4.08...$$

$$= 10.049...$$

Green Lune

$$\text{Sector area} = \frac{106.26\dots}{360} \times \pi \times 5^2 = 23.18\dots$$

$$\text{Triangle area} = \frac{1}{2} \times 5 \times 5 \times \sin(106.26\dots) = 12$$

(using sine rule)

$$\text{Segment area} = 23.18 - 12 = 11.18\dots$$

Line BC is diameter because angles in semicircle = 90° (Thales Theorem)

$$\text{Semicircle} = \frac{1}{2} \times \pi \times 4^2 = 25.13\dots$$

$$\begin{aligned} \text{Lune} &= \text{semicircle} - \text{segment} \\ &= 25.13\dots - 11.18\dots \\ &= 13.95\dots \end{aligned}$$

$$\text{Total lune area} = \underset{\text{area}}{\text{green lune}} + \underset{\text{area}}{\text{blue lune}}$$

$$= 10.049\dots + 13.95\dots$$

$$= 24$$