Equation or Identity.

Kyle Settrey.

$$\sin\left(\frac{\pi}{2} - A\right) = \cos A$$

$$\sin\frac{\pi}{2}\cos A - \sin A\cos\frac{\pi}{2} = \cos A$$

$$1 \times \cos A - \sin A \times 0 = \cos A$$

$$\cos A = \cos A$$

$\therefore$ it is an identity

$$\sin\left(\frac{\pi}{2} - A\right) \equiv \cos A$$

$$\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$$

$$\sin\left(\frac{\pi}{2} - A\right) = \sin\frac{\pi}{2}\cos A - \sin A\cos\frac{\pi}{2}$$

$$\sin\frac{\pi}{2} = \cancel{0}1 \qquad \cos\frac{\pi}{2} = 0.$$

$$\sec^2 A - \tan^2 A = 1$$

$$\left(\frac{1}{\cos A}\right)^2 - \left(\frac{\sin A}{\cos A}\right)^2 = 1$$

$$\frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A} = 1$$

$$\frac{1 - \sin^2 A}{\cos^2 A} = 1$$

$$\frac{\cos^2 A}{\cos^2 A} = 1$$

$$1 = 1$$

$$\sec A = \frac{1}{\cos A} \qquad \tan A = \frac{\sin A}{\cos A}$$

$\because \sin^2 A + \cos^2 A = 1$

$1 - \sin^2 A = \cos^2 A.$

$\therefore$ it is an identity $\because$ both sides are proven to be exactly equal.

$$\sec^2 A - \tan^2 A \equiv 1$$

Equation or Identity cont. Kyle Settrey.

$\tan(A+B) = -\tan C$

A, B, C are angles in a triangle

$\therefore A+B+C = 180$ $\because$ angles in a triangle add to 180.

$\therefore C = 180-(A+B)$

$\tan(A+B) = -\tan(180-(A+B))$

Let $(A+B) = D$

$\tan(A+B) = -\tan(180-D)$

$\tan(A\pm B) = \dfrac{\tan A \pm \tan B}{1 \pm \tan A \tan B}$

$\tan(A+B) = -\dfrac{\tan 180 - \tan D}{1 - \tan 180 \tan D}$

$\tan 180 = 0.$

$\tan(A+B) = -\dfrac{0-\tan D}{1 - 0 \times \tan D}$

$\tan(A+B) = -\dfrac{-\tan D}{1}$

$\tan(A+B) = --\tan D$

$\tan(A+B) = \tan D$

$D = (A+B)$

$\tan(A+B) = \tan(A+B)$

$\therefore$ it is an identity $\because$ both sides are proven to be exactly equal.

$\tan(A+B) \equiv -\tan C.$

 Kyle Settrey

$\sin^2 A - \cos^2 B = 1$

$0 \leq \sin^2 A \leq 1$
$0 \leq \cos^2 B \leq 1$ always true.

$\because$ $\cos^2 B$ cannot be negative and $\sin^2 A$ canot be larger than 1, it can be concluded that $\sin^2 A = 1$ and $\cos^2 B = 0$ for equation to be true.
It can also here be concluded that the equation is not an identity $\because$ it only works for specific values of A and B.

A, B, C are angles in a triangle
$\therefore$ $0 < A, B, C < 180$
$A + B + C = 180$

~~sin~~ $\because$ ~~$0 < A, < B$~~ $0 < A < 180$, for $\sin^2 A = 1$, $A = 90°$
also $\because$ $0 < B < 180$, for $\cos^2 B = 0$, $B = 90°$

since $A + B + C = 180$ $C = 0$ if $A, B = 90$ $\because$ $C \neq 0$
$\therefore$ it does not work for any triangle since $C \neq 0$ but must $= 0$ to work, there are no other values for A, B that work between 0 and 180.

$\sin^2 A - \cos^2 B = 1$ can work, <u>but</u> does not work for any triangle.

Equation or Identity cont..

Kyle Sertrey

$\sin(\pi - A) = \sin B$

$\sin\pi\cos A - \sin A\cos\pi = \sin B$

$0 \times \cos A - \sin A \times -1 = \sin B$

$\sin A = \sin B$

$\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$

$\sin(\pi - A) = \sin\pi\cos A - \sin A\cos\pi$

$\cos\pi = -1 \qquad \sin\pi = 0$

$\sin A = \sin B$ when $A+B = 180$ or when $A = B$

$\because$ it is a triangle $A+B < 180$

so equation only works when $A = B$

Equations or Identity cont.   Kyle Settney.

$$\sin^2 A - 3\cos A = \text{cosec}^2 C - \cot^2 C$$

$\text{cosec}\, C = \frac{1}{\sin C}$

$$\sin^2 A - 3\cos A = \left(\frac{1}{\sin C}\right)^2 - \left(\frac{1}{\tan C}\right)^2$$

$\cot C = \frac{1}{\tan C}$

$$\sin^2 A - 3\cos A = \frac{1}{\sin^2 C} - \frac{1}{\tan^2 C}$$

$\tan^2 C = \frac{\sin^2 C}{\cos^2 C}$

$$\sin^2 A - 3\cos A = \frac{1}{\sin^2 C} - \frac{1}{\frac{\sin^2 C}{\cos^2 C}}$$

$$\sin^2 A - 3\cos A = \frac{1}{\sin^2 C} - \frac{\cos^2 C}{\sin^2 C}$$

$$\sin^2 A - 3\cos A = \frac{1 - \cos^2 C}{\sin^2 C}$$

$\sin^2 C + \cos^2 C = 1$

$1 - \cos^2 C = \sin^2 C$

$$\sin^2 A - 3\cos A = \frac{\sin^2 C}{\sin^2 C}$$

$$\sin^2 A - 3\cos A = 1$$

$$1 - \cos^2 A - 3\cos A = 1$$

for equation to work $\cos^2 A + 3\cos A = 0$

so solve for A

$\cos A(\cos A + 3) = 0$

so $\cos A = 0$ or $\cos A = -3$

$\cos A$ must $= 0$ since $\cos A = -3$ is not a value

so equation only works when $A = 90°$ $\because$ $0 < A < 180$ due to angles in a triangle