

Equal Means - Stage: 3 - Short Challenge Level: ★★

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We are given the following list of numbers and asked 'What value of x makes the mean of the first three numbers in this list equal to the mean of the last four?'. The first three numbers are: 15, 5, and x , whilst the last four numbers are: x , 7, 9, and 17

$$15 \quad 5 \quad x \quad 7 \quad 9 \quad 17$$

To calculate the mean of a set of numbers, you firstly find out the total (by adding up each value) and dividing by the number of values thus the mean of the first three numbers is as follows:

$$\frac{15 + 5 + x}{3}$$

And, the mean of the last four numbers is as follows:

$$\frac{x + 7 + 9 + 17}{4}$$

We are told that the mean of the first three is equal to the mean of the last four thus, the following is true:

$$\frac{15 + 5 + x}{3} = \frac{x + 7 + 9 + 17}{4}$$

We can collect like terms to deduce the following:

$$\frac{20 + x}{3} = \frac{x + 33}{4}$$

To remove the denominator, we must cross-multiply or multiply both sides by the LCM (lowest common multiple). The LCM of 3 and 4 is 12. Once we have removed the denominator our equation looks like the following:

$$4(20 + x) = 3(x + 33)$$

We can then multiply out the brackets, multiplying both terms within the brackets by the relevant number.

$$80 + 4x = 3x + 99$$

Utilising general algebraic rules and knowledge, we can isolate the term of x and deduce its value.

$$80 + 4x = 3x + 99$$

Subtract 80 from both sides:

$$4x = 3x + 19$$

Subtract $3x$ from both sides:

$$x = 19$$

We can then place it back in our first equation to assert that 19 is the correct value of x .

$$\frac{15 + 5 + 19}{3} = \frac{19 + 7 + 9 + 17}{4}$$

$$\frac{39}{3} = \frac{52}{4}$$

$$13 = 13$$

Thus, the value of x is 19, and the ending set of values is as follows:

15 5 19 7 9 17