Do Rare Events Happen: Stage 3 and 4** Zach Thompson

When I first read the question, I thought that this would be a really rare event, but then realised this is probably more common than I think.

I've never done this type of maths before. The question is very open and requires a different way of thinking.

Initially, I started by assuming that the parent(s) were able to have children, able to continue to do so (after the first child and second child) and successfully deliver a live birth. But these assumptions are already factored into the statistic of 1 million families in the UK having at least 3 children (because logically these assumptions must be true if the family has at least 3 children), but I at least started with that thought.

Next I assumed that a child could be born on any day of the year, and had an equal chance of doing so. This is a reasonable assumption, the question does not specify that the first child had to be born on a specific day, (which would reduce the probability considerably), only that the second and third births must be on the same calendar day (but not year) as the first. This means that the first birth can be completely random. The only certainty (in this question) is that the first child will be born at some point during a year.

My final assumption was to ignore leap years and set the number of days in a year as 365. Again, this is reasonable, given that leap years occur once every four years, so the 29th February will only appear approximately 25 times in 100 years (this can also be 24, but 25 is a reasonable approximation as it is probably most peoples' understanding).

So: $\frac{25}{(75\times365)+(25\times366)} = \frac{1}{1461} = 0.000684$, which is not impossible, but is rather improbable.

I stuck with these assumptions, because if I tried to put in every factor, it would become too complex, as all I really wanted was a reasonable idea of how likely it is for three siblings to share the same birthday

The first child can be born on any day, so the probability is $\frac{365}{365}$ (it will be born, we just don't know when).

The probability of the second child being born on exactly the same day is $\frac{1}{365}$, because you want the second birthday to match the first.

The probability of the third child being born on exactly the same day is also $\frac{1}{365}$, because again, you want the third birthday to match the first and second birthday.

We can now combine the probabilities:

$$\frac{365}{365} \times \frac{1}{365} \times \frac{1}{365} = \frac{1}{133225}$$

If there are 1,000,000 families in the UK with at least 3 children, then:

$$\frac{1}{133225} \times 1000000 = 7.506$$

You cannot have 7.5 families, but there is a probability that there are 7 or so families in the UK with this phenomena – which is amazing!

Extension

The probability of all three births occurring on a specific given date e.g. 9th July (my birthday) is much, much smaller, as the first child must also be born on a specific day, rather than a random one. This alters the calculation to:

$$\frac{1}{365} \times \frac{1}{365} \times \frac{1}{365} = \frac{1}{48627125} \times 1000000 = 0.02056$$

This is extremely unlikely and probably what most people think when they first read the question.

You can also then calculate the probability of 3 siblings sharing a birthday and all being boys. Although I read somewhere that more boys are born than girls (can be 108:100), I will assume a standard ratio of 50:50, because this is most peoples' understanding.

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \times 1000000 = 125000$ families with three boys

To find out how many of these families all share the same birthday, just combine the probabilities:

$$\frac{1}{365^3} \times \frac{1}{8^3} = 2.57 \times 10^{-9} \times 1000000 = 0.00257$$

Which is extremely improbable, although that doesn't mean it's not possible.

Final thought: different countries have different populations and family sizes. This would affect the probabilities.