$\cancel{n} - n\lambda + \lambda - 1 = \cancel{n} - \lambda n^2$

$\lambda n^2 - n\lambda + \lambda - 1 = 0$

$\lambda n^2 - n\lambda + \lambda = 1$

$\lambda(n^2 - n + 1) = 1$

$\lambda = \dfrac{1}{n^2 - n + 1}$

$\therefore$ MP is $\dfrac{1}{n^2-n+1}$ th of MB → we can rotate the triangle and yet again prove that TR is $\dfrac{1}{n^2-n+1}$ th of TO and NQ is $\dfrac{1}{n^2-n+1}$ th of AN.

Next look at ΔABM let

~~ΔABM is $\frac{1}{3}A_1$ because it has the same base as ΔOAB but a~~

area ΔABM is $\frac{1}{n}A_1$ because it has the same base ΔOAB ie (AB) but $\frac{1}{n}$ th its height.

ΔAPM has the base as ΔABM ~~with~~ (ie AM) but ΔAPM has $\dfrac{1}{n^2-n+1}$ th the height.

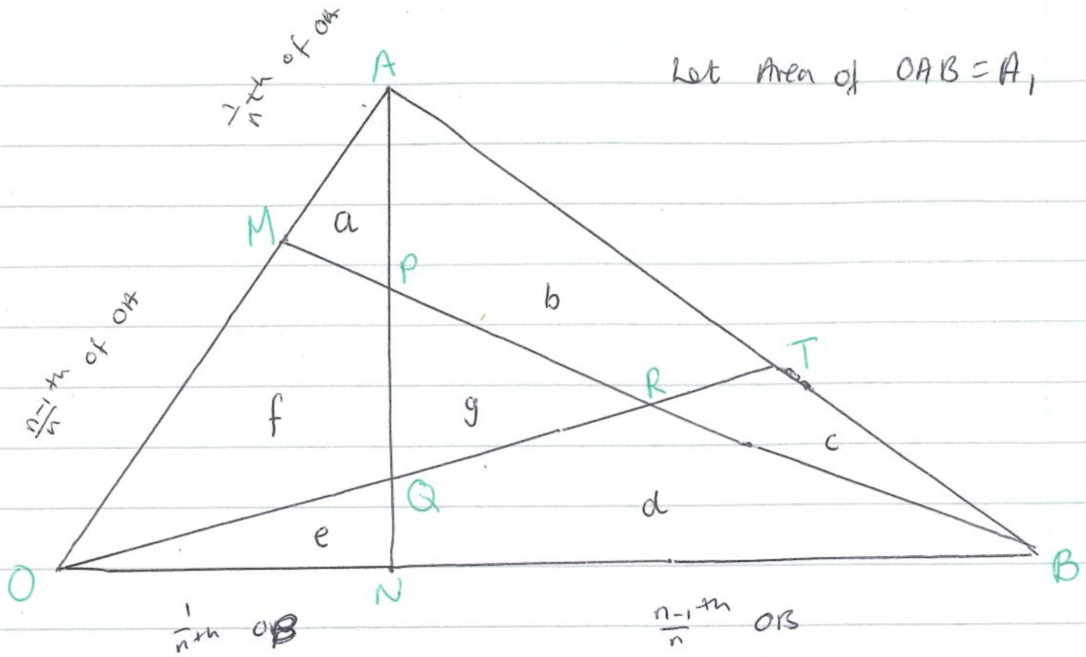
$\therefore \Delta APM$ has area $= \dfrac{1}{n^2-n+1} \times$ area of ΔABM

$= \dfrac{1}{(n^2-n+1)} \cdot \dfrac{1}{n} A_1$

$a = \dfrac{1}{n(n^2-n+1)} A_1$

~~but a = e = c~~ but $a+b+c = e+d+c$

$\therefore a = e = c = \dfrac{1}{n(n^2-n+1)} A_1$



Let Area of $OAB = A_1$

Consider the triangle above. ~~OA~~ AM is $\frac{1}{n}$th of the way along AO, BT is $\frac{1}{n}$th of the way along BA and ON is $\frac{1}{n}$th of the way along OB. What fraction of ΔOAB is the area g?

let $\vec{OA} = \underline{a}$ and let $\vec{OB} = \underline{b}$

We will ~~try~~ now try and find two ways of expressing $\vec{OP}$.

$\vec{OP} = \vec{OM} + \vec{MP}$

$\vec{OP} = \frac{n-1}{n}\underline{a} + \lambda(\vec{MB})$

$= \frac{n-1}{n}\underline{a} + \lambda(-\frac{n}{n-1}\underline{a} + \underline{b})$

$= \frac{n-1}{n}\underline{a} - \frac{n}{n-1}\lambda\underline{a} + \lambda\underline{b}$

$= \frac{n-1}{n}(1-\lambda)\underline{a} + \lambda\underline{b}$

$\vec{OP} = \vec{ON} + \vec{NP}$

$= \vec{ON} + \mu(\vec{NA})$

$= \frac{1}{n}\underline{b} + \mu(-\frac{1}{n}b + a)$

$= \frac{1}{n}\underline{b} - \frac{1}{n}\mu\underline{b} + \mu\underline{a}$

$= \frac{1}{n}(1-\mu)\underline{b} + \mu\underline{a}$

Comparing the vector coefficients

$\frac{n-1}{n}(1-\lambda) = \mu$

$(n-1)(1-\lambda) = \mu n$

$n - n\lambda + \lambda - 1 = \mu n$ ①

$\frac{1}{n}(1-\mu) = \lambda$

$1 - \mu = \lambda n$ ~~②~~

$1 - \lambda n = \mu$ ②

② into ① $\quad n - n\lambda + \lambda - 1 = (1-\lambda n)n$

$$\frac{(n-1)(n^2-n+1) - (2n^2-2n-1)}{n(n^2-n+1)}$$

$$= \frac{n(n^2-n+1) - (n^2-n+1) - 2n^2+2n+1}{n(n^2-n+1)}$$

$$= \frac{n(n^2-n+1) - n^2+n-1-2n^2+2n+1}{n(n^2-n+1)}$$

$$= \frac{n(n^2-n+1) - 3n^2+3n}{n(n^2-n+1)}$$

$$= \frac{n(n^2-n+1) - 3n(n-1)}{n(n^2-n+1)}$$

$$= \left(1 - \frac{3n(n-1)}{n(n^2-n+1)}\right) A_1$$

This gives a general solution ~~Since for a~~ for the fraction of the area that g takes up if each side is marked $\frac{1}{n}$th of the way across a line.

At $n=3$, ~~then~~ we have our original probe.

$$\therefore g = \left(1 - \frac{3\cdot 3(2)}{3(3^2-3+1)}\right) A_1$$

$$= \left(1 - \frac{3\cdot 3\cdot 2}{3\cdot 7}\right) A_1$$

$$= \left(1 - \frac{18}{21}\right) A_1$$

$$= \left(1 - \frac{6}{7}\right) A_1$$

$$= \frac{1}{7} A_1$$

Some other properties that we proved along the way are that $a \equiv c \equiv e$ and $b \equiv f \equiv d$

Looking at ΔANB

ΔANB has the same base as ΔOAB (ie AB) but it has a ~~base~~ height $\left(\frac{n-1}{n}\right)$th of ΔOAB.

$\therefore \Delta ANB$ has $\frac{n-1}{n} \times$ the area of ΔAOB

area $\Delta ANB = \frac{n-1}{n} A_1$

~~area $\Delta ANB = g+b+c+d$~~ $\quad \boxed{g+b+c+d = \frac{n-1}{n} A_1}$

~~$\therefore g = \text{area } \Delta ANB - (b+c+d)$~~

~~Also~~ But $b = \text{area } \Delta ABM - a - c$

$$= \frac{1}{n} A_1 - \frac{1}{n(n^2-n+1)} A_1 - \frac{1}{n(n^2-n+1)} A_1$$

~~Also~~ $$= A_1 \left(\frac{1}{n} - \frac{2}{n(n^2-n+1)}\right)$$

$$= A_1 \left(\frac{n^2-n+1}{n(n^2-n+1)} - \frac{2}{n(n^2-n+1)}\right)$$

$$= A_1 \left(\frac{n^2-n+1-2}{n(n^2-n+1)}\right)$$

$$b = A_1 \left(\frac{n^2-n-1}{n(n^2-n+1)}\right)$$

in the same way $d = \left[\frac{n^2-n-1}{n(n^2-n+1)}\right] A_1 = b$

$\therefore$

$$g+b+c+d = \frac{n-1}{n} A_1$$

$$g+2b+c = \frac{n-1}{n} A_1$$

$$g = \left(\frac{n-1}{n}\right) A_1 - 2b - c$$

$$= \left(\frac{n-1}{n}\right) A_1 - (2b+c)$$

$$= \left(\frac{n-1}{n}\right) A_1 - \left(\frac{2\cdot(n^2-n-1)}{n(n^2-n+1)} A_1 + \frac{1}{n(n^2-n+1)} A_1\right)$$

$$= \frac{n-1}{n} - \left(\frac{2n^2-2n-2+1}{n(n^2-n+1)}\right)$$