

(LET B be any point within rectangle ADCE)

Since quadrilateral ADCE is a rectangle, AD is parallel to CE, and so there will always be a line that can be drawn through point B which will be parallel with both AD and CE (dashed line).

Therefore, since line XY is parallel to AD, $\angle DAB$ and $\angle ABX$ are alternate.

$$\therefore \angle DAB \text{ ("green" angle)} = \angle ABX$$

Also, line XY is parallel to CE, so $\angle BCE$ and $\angle CBX$ are alternate.

$$\therefore \angle BCE \text{ ("blue" angle)} = \angle CBX$$

However, the "red" angle CBA is made up of $\angle CBX$ and $\angle ABX$.

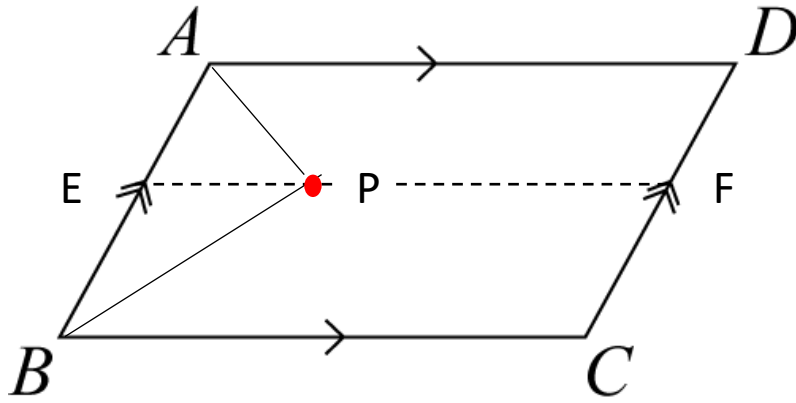
$$\angle CBA = \angle CBX + \angle ABX$$

Substituting values of $\angle CBA$ and $\angle ABX$ into the equation,

$$\angle CBA = \angle DAB + \angle BCE$$

\therefore the "red" angle = the "green" angle + the "blue" angle Q.E.D

(NB if the point goes outside the rectangle, the same theory applies. The "extended" dotted line would still be parallel to AD and CE, so the red angle would still maintain its composition as the sum of the green angle and the sum of the blue angle)



The same principle applies for a parallelogram (I will use a different proof).

LET angles DAP, APB and PBC be a , b and c respectively.

Since AD is parallel to BC, angles DAB and ABC are allied.

$$\therefore \angle DAB + \angle ABC = 180^\circ$$

$$\angle DAB = \angle DAP + \angle PAB$$

$$= a + \angle PAB$$

Also, $\angle ABC = \angle PBC + \angle ABP$

$$= c + \angle ABP$$

Therefore, using $\angle DAB + \angle ABC = 180$,

$$a + \angle PAB + c + \angle ABP = 180^\circ.$$

Hence, the two angles PAB and ABP in triangle APB add up to $(180 - a - c)^\circ$.

However, since angles in triangle APB add up to 180, the two angles PAB and ABP also sum to $(180 - b)^\circ$.

We can now equate the two:

$$(180 - a - c) = (180 - b)$$

$$\therefore a + c = b \text{ (as with the rectangle)} \quad \text{Q.E.D}$$

So yes, Alison's conjecture is true.