

N= number of days
T= triangle number

Option 1

Option 1 is relatively simple. It says that every day, 100 dollars is given, which means that the money received per day would be at a constant 100, and the cumulative equation would be $100 * n$.

Option 2

$25 + 5n$ = per day equation

We start off at 30, and add an extra \$5 per day.

2nd day: $30+5$

3rd day: $30+2(5)$

So,

Nth day= $30+5(n-1)$

Which can be simplified to:

$25 + 5n$

$25n + 5 * T(n)$ = cumulative equation

This is where triangle numbers come in, which are basically sums of numbers ($1+2+3+4+...+n$). To solve this, we merge the numbers into pairs, $1+n$, $2+(n-1)$ and so on. They all add up to $n+1$. This pattern goes on until we hit the median number, similar to the way common factors hit a turning point. Therefore, the expression for triangle numbers are $(n*(n+1)/2)$.

However, since the amount of pocket money increases by 5 (so 5, 10, 15, 20 instead of 1, 2, 3, 4), we have to times the triangle number expression by 5, and times the base amount of money by n , since we are finding the cumulative amount. Hence, we reach:

$25n + 5 * (n(n + 1)/2)$

Option 3

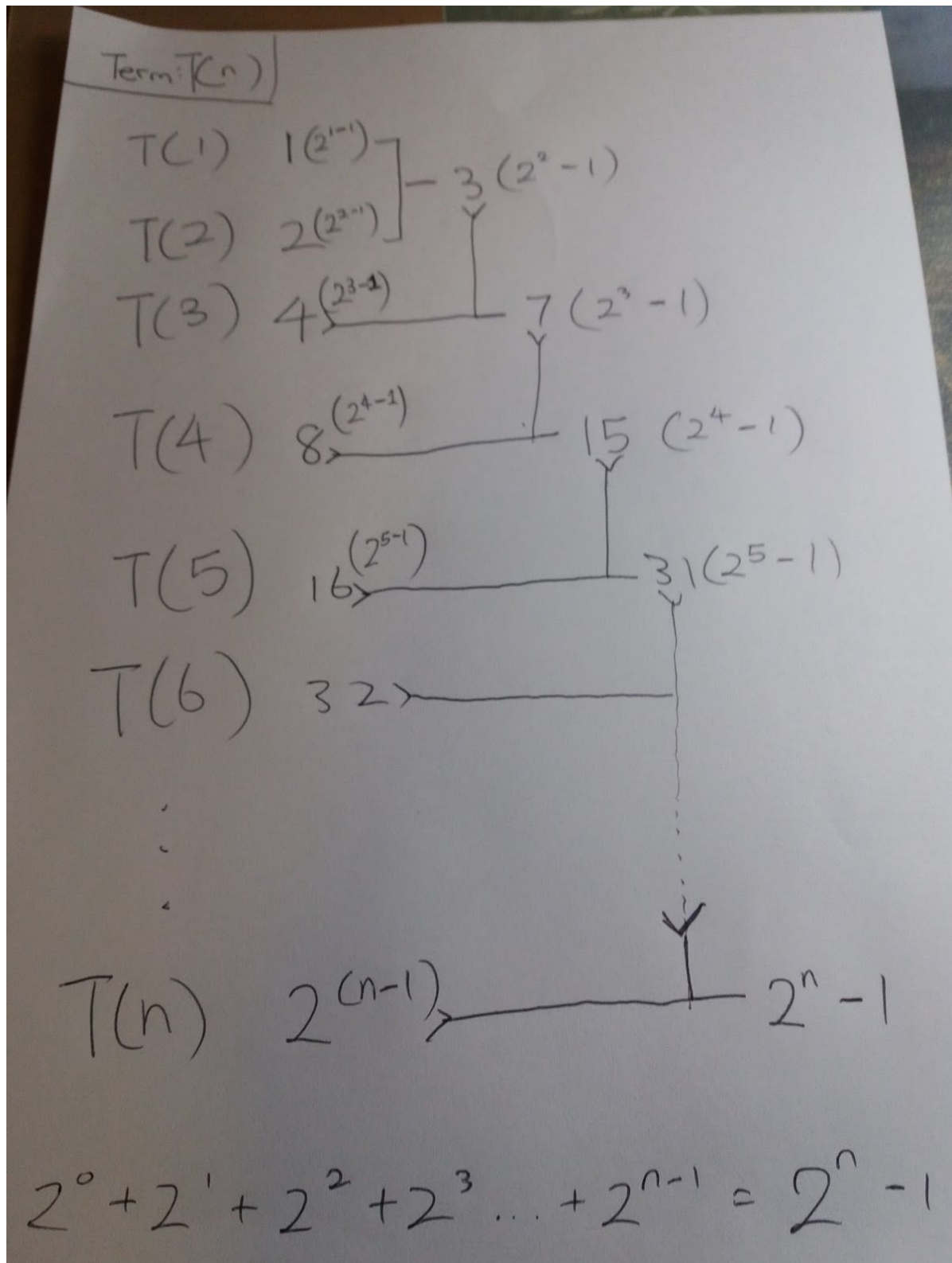
This is perhaps the hardest one to come up with a general formula for. If you look at the amount of money you get per week, they are the powers of 2 divided by 10. This means that on day n , you get $\frac{2^{n-1}}{10}$ dollars (it is n because you start at 0.1, not 0.2).

The cumulative equation for option 3: $(2^n - 1)/10$

1st of all, boiling down this problem, it comes down to this: what is the triangle number for $2^0 + 2^1 + 2^2 + 2^3 ... + 2^n$?

Proof:

*Starting with 1 for easier understanding. In the case of option 3, just divide by 10.



Reasoning:

Why is this, you may ask. To find out why, let us first look at summing up powers of 2 in binary. Each power of 2 is a 1 followed by 0s ($1 = 1$, $2 = 10$, $4 = 100$, $8 = 1000$ *etc.*) As you move on to the next power, you add one more zero. 2^n will be a 1 followed by n 0s. This means, that when you sum up the powers of 2 (starting with 1), you get a string of 1s ($1 + 2 + 4 = 7 = 1 + 10 + 100 = 111$, $1 + 2 + 4 + 8 = 1 + 10 + 100 + 1000 = 1111$). Such strings are always 1 less than a power of 2. If you add 1 to such a string, you will get a 1 followed by $n+1$ 0s (where n is the power you are summing to).

Therefore, summing up powers of 2 up to 2^n gives $2^{n+1} - 1$

The reason for the differences in the cumulative equation for the pocket money example are as follows:

Because you start at day 1, not day 0, n becomes $n - 1$ so 2^{n+1} becomes 2^n

The reason for dividing by 10, is because you start at 0.1, not 1, so each day's income is a tenth of 2^{n-1}