

NOTE: £1 = 10HKD (approx.) so original problem is in a different currency.

Option 1

For the first sequence, the amount of money received each day is \$100. This means that the common difference between the total amount of money possessed each day is 10. So it goes like this -
10, 20, 30, 40, 50, 60, 70, 80, 90, 100...

It is a linear sequence with an equation of $10d$. As we can see the first difference is already always the same, so it has no need for a second difference. This means that it is linear and isn't a quadratic sequence.

Compared to the other options, this option is the most viable option for the first 13 days. With 130 pounds it is vastly greater than the other two options. The other two on day 13 are 78 and 81.91. Option 1 stays ahead of option two until day 29.

Option 2

For the second sequence, the order of the numbers/money received each day increases by a common difference, which is 0.5 dollars. It is an arithmetic sequence, and the amount received each day can be shown in a sequence of -

3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8....

As we can see, the common difference between the terms next to each other is 0.5. In order to find out how effective the money plan is, we need to find out the total sum of the money given in the month (in this case, we will use 30 days to make it easier) so that we can compare it with the other options.

The equation for the total sum of an arithmetic sequence is (where from?)

$$S_n = \frac{n}{2} (a_1 + a_n)$$

However, we do not know what “An” is, and we must input another equation into this formula. “An” can be expressed as $a_n = a_1 + (n - 1)d$.

By inputting this equation in “Sn”, we can find the sum of the sequence with the equation

$$S_n = n/2 [2a_1 + (n-1)d]$$

(in this case, n being the total number of days (30), a1 being the first number/amount (3), and “d” being the common difference (0.5))

So therefore, $S_n = 30/2 [2(3) + (30-1)0.5]$, which gives **307.5 pounds after 30 days**. This equation can also be used to calculate the sum of money given after any given day.

We see that this pocket money plan only overtakes option 1 as the month ends, so it is unwise to choose this option in February. Also, compared to option 3, 1 and 2 have a lot less in total after a whole month. However, if you were to choose one after 8 days, it would not be wise to choose option 3.

Option 3

Option 3:

- S_n : sum
- r : multiplier (2)
- s : initial amount (0.1)
- n : n^{th} term

$$S_n = s \times r + s \times r^2 + s \times r^3 \dots s \times r^n$$

$$S_n \times r = s \times r^2 + s \times r^3 + s \times r^4 \dots s \times r^{n+1}$$

$$S_n \cdot r - S_n = s r^n - s r$$

$$S_n (r - 1) = s r (r^{n-1} - 1)$$

$$S_n = \frac{s r (r^{n-1} - 1)}{r - 1}$$

Option 3 is an arithmetic series, which means that in this sequence instead of having a common difference there is a common ratio between the terms. The sequence would be -
0.01, 0.02, 0.04, 0.08, 0.16, 0.32....

Again, to see how effective it is, we have to find out the total sum of money earned after a month, which can be expressed as -

$$S_n = a_1(1-r^n)/(1-r)$$

In this case, a_1 is once again the starting number in the sequence, which would be 0.01. "R" would be the common ratio, which is 2, as each term next to each other is either 2 times larger or smaller. N would be the number of days, in this case we could use 30 to represent one month.

Therefore, the calculation would be -

$$S_n = 0.01 (1 - 2^{30}) / 1 - 2$$

Which would give a total of 10,737, 418.23 pounds, which, compared to the other options after one month, is certainly the most desirable as it gives you the largest amount of money.

Which option?

3, but only if it is over a complete month (but it could be any month).

In which months would option 1 be better than option 2?

February, but only when it is not a leap year.

If your family stopped your pocket money on day 8, which option would give you the most?

Option 1.

On which day of the month does option 3 become the most fruitful?

The last day for every month?

If you chose option 3, how many days would it be before you became a millionaire?

After day 27.