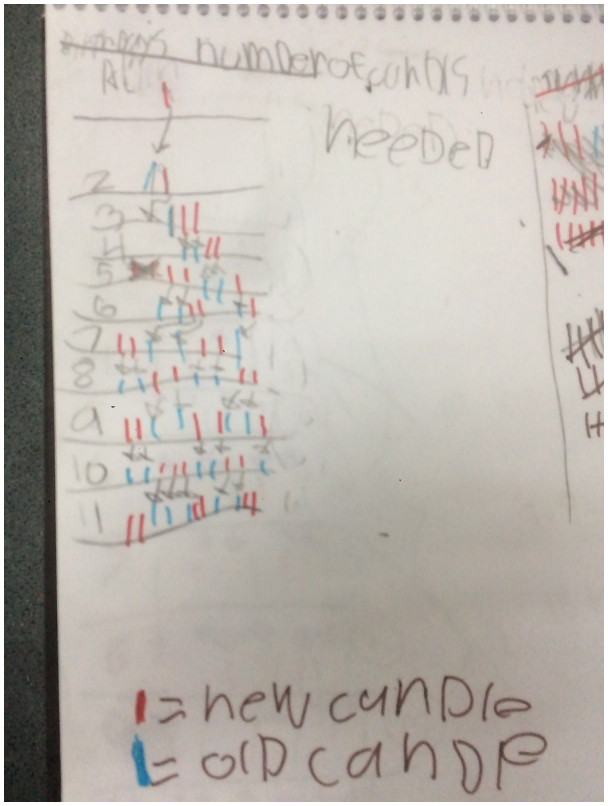
Solution:

If candles last less than a year, Ben's parents will need 66 birthday candles for Ben's cakes, up to and including his eleventh birthday.

I know this because I added up the number of candles his parents would need to buy each year: 1 + 2 + 3 + 4 + 5+ 6 + 7 + 8 + 9+ 10 +11 = 66.

Next, I found out how many candles he would need if they lasted exactly two years.

I found this out by creating a color-coded tally chart (see picture).



So if Candles Last 2 years...

Birthday	New Candles Needed
1st	1
2nd	1
3rd	2
4th	2

5th	3
6th	3
7th	4
8th	4
9th	5
10th	5
11th	6

Then, I added up all of the new candles needed. If the candles last two years, they would need 36 candles.

If Candles Last 3 years...

Birthday	New Candles Needed
1st	1
2nd	1
3rd	1
4th	2
5th	2
6th	2
7th	3
8th	3
9th	3
10th	4
11th	4

I used the same strategy to find out that he needed 26 candles if the candles last three years.

Birthday	New Candles Needed
1st	1
2nd	1
3rd	2
4th	2
5th	3
6th	3
7th	4
8th	4
9th	5
10th	5
11th	6

Then I noticed a pattern! I noticed that if the candles last 1 year, the number of new candles is not repeated, but if the candles last 2 years, you have to buy 1 candle for 2 years and 2 candles for 2 years, and 3 candles for 2 years, and 4 candles for 2 years and so on. If the candles last 3 years, you have to buy 1 candles for 3 years and 2 candles for 3 years, and 3 candles for 3 years and so on.

Then, I used the pattern to calculate the number of candles needed for the rest of the years. For example, if the candles last 3 years, I used this equation: 1 + 1 + 1 + 2 + 2 +2 + 3 + 3+ 3 + 4 + 4 = 26

Years Candle Last	Expression Used to Find Number of Candles Needed	Total Number of Candles Needed
1	1 + 2 + 3 + 4 + 5+ 6 + 7 + 8 + 9+ 10 +11	66
2	1 + 1 + 2 + 2 + 3 +3 + 4 + 4+ 5 + 5 + 6	36

3	1 + 1 + 1 + 2 + 2 +2 + 3 + 3+ 3 + 4 + 4	26
4	1 + 1 + 1 + 1 + 2 +2 + 2 + 2+ 3 + 3 + 3	21
5	1 + 1 + 1 + 1 + 1 +2 + 2 + 2+ 2 + 2 + 3	18
6	1 + 1 + 1 + 1 + 1 +1 + 2 + 2+ 2 + 2 + 2	16
7	1 + 1 + 1 + 1 + 1 +1 + 1 + 2+ 2 + 2 + 2	15
8	1 + 1 + 1 + 1 + 1 +1 + 1 + 1+ 2 + 2 + 2	14
9	1 + 1 + 1 + 1 + 1 +1 + 1 + 1+ 1 + 2 + 2	13
10	1 + 1 + 1 + 1 + 1 +1 + 1 + 1+ 1 + 1 + 2	12
11	1 + 1+ 1 + 1+ 1+ 1+ 1+ 1+ 1+ 1+ 1	11