1. From the diagram we can say

$$\frac{x}{2} = \frac{y}{4}$$

$$\Rightarrow x = 2y$$

$$\Rightarrow x^2 = 4y^2$$

This would mean 4 squares with side length y could fit into 1 square of side length x.

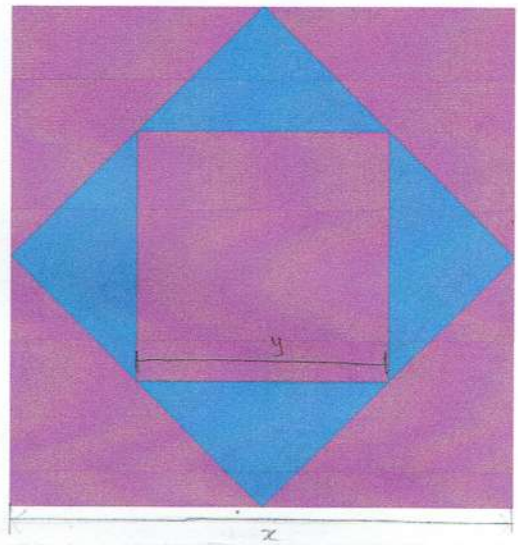
$$\Rightarrow x^2 = \frac{1}{4} y^2$$

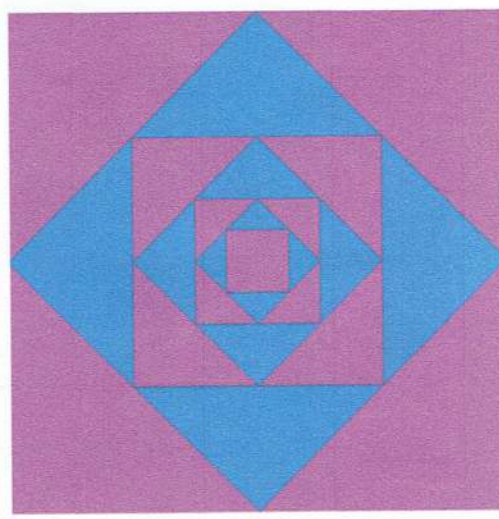
or every successive purple square is a quarter of the previous purple square.

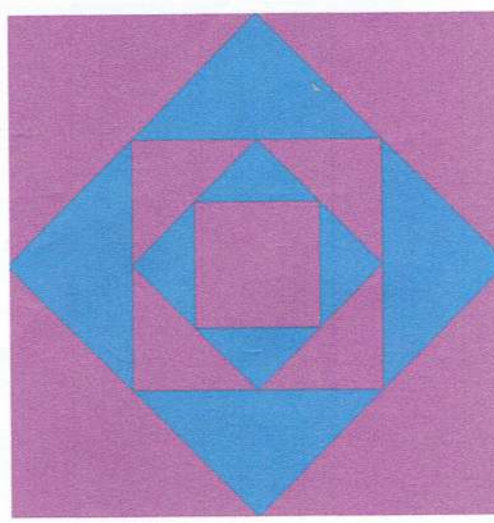
We can use this information to create a sequence:

$$\frac{1}{4}, \frac{1}{16}, \frac{1}{64} \ldots \frac{1}{4^n}$$

We can use this to find the proportion of a certain number of squares, or to infinity.







$$\sum_{n=0}^{n} \left(\frac{1}{4^n}\right) = \frac{a(1-r^n)}{1-r}$$

(Sum to a certain number of squares, n = number of squares from the original outer purple square)

n = 0 for outer square

$$= \frac{\left(\frac{1}{4}\right)\left(1-\left(\frac{1}{4}\right)^n\right)}{1-\left(\frac{1}{4}\right)}$$

$$= \frac{\left(\frac{1}{\cancel{4}}\right)\left(1-\left(\frac{1}{4}\right)^n\right)}{\left(\frac{3}{\cancel{4}}\right)}$$

$$= \frac{4^n-1}{3\cdot 4^n}$$

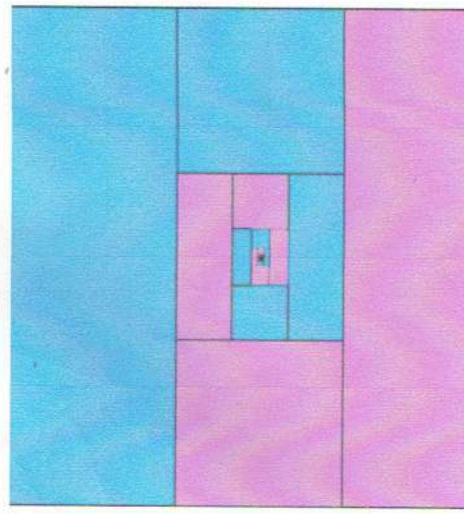
$\Rightarrow$ proportion of blue $= 1-\left(\dfrac{4^n-1}{3\cdot 4^n}\right)$

$$\sum_{n=1}^{\infty} = \frac{a}{1-r} = \frac{\left(\frac{1}{4}\right)}{1-\left(\frac{1}{4}\right)} = \frac{\left(\frac{1}{\cancel{4}}\right)}{\left(\frac{3}{\cancel{4}}\right)} = \frac{1}{3}.$$

Steps for the problem:

1. Find a relation (common ratio)
2. Make a sequence
3. Find the sums

2. Straight away we can see the square is always being split into 3 parts.



$\Rightarrow r = \frac{1}{3}$

The sequence would be :

$$\frac{1}{3}, \frac{1}{9}, \frac{1}{27} \dots \frac{1}{3^n}$$

$$\Rightarrow \sum_{n=1}^{n}\left(\frac{1}{3^n}\right) = \frac{a(1-r^n)}{1-r}$$

$$= \frac{\left(\frac{1}{3}\right)\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\left(\frac{1}{3}\right)}$$

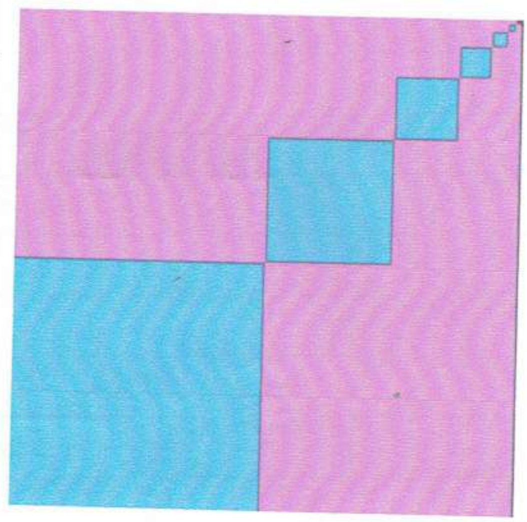
$$= \frac{\left(\frac{1}{\cancel{3}}\right)\left(1-\left(\frac{1}{3}\right)^n\right)}{\left(\frac{2}{\cancel{3}}\right)}$$

$$= \frac{3^n - 1}{2 \cdot 3^n}$$

(No 1 - purple beca
they are exactly the s

$$\sum_{n=1}^{\infty}\left(\frac{1}{3^n}\right) = \frac{a}{1-r} = \frac{\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)} = \frac{\left(\frac{1}{\cancel{3}}\right)}{\left(\frac{2}{\cancel{3}}\right)} = \frac{1}{2}$$

3. Straight away you can tell that the successive square is a quarter of the previous square.



$r = \frac{1}{4}$

We can create a sequence:

$$\frac{1}{4}, \frac{1}{16}, \frac{1}{64} \ldots \frac{1}{4^n}$$

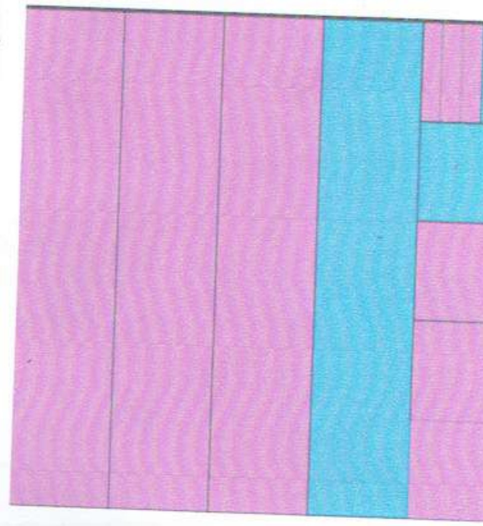
$$\sum_{n=1}^{n}\left(\frac{1}{4^n}\right) = \frac{a(1-r^n)}{1-r}$$

$$= \frac{\left(\frac{1}{4}\right)\left(1-\left(\frac{1}{4}\right)^n\right)}{1-\left(\frac{1}{4}\right)}$$

$$= \frac{4^n-1}{3\cdot 4^n}$$ (as in the first example)

$$\sum_{n=1}^{\infty} = \frac{a}{1-r} = \frac{\left(\frac{1}{4}\right)}{1-\left(\frac{1}{4}\right)} = \frac{\left(\frac{1}{\cancel{4}}\right)}{\left(\frac{3}{\cancel{4}}\right)} = \frac{1}{3}$$

4. We can see that the square is continually cut up into 5 pieces.



$\Rightarrow r = \frac{1}{5}$.

$\Rightarrow \frac{1}{5}, \frac{1}{25}, \frac{1}{125} \ldots \frac{1}{5^n}$

$$\Rightarrow \sum_{n=1}^{n} = \frac{a(1-r^n)}{1-r}$$

$$= \frac{\left(\frac{1}{5}\right)\left(1-\left(\frac{1}{5}\right)^n\right)}{1-\left(\frac{1}{5}\right)}$$

$$= \frac{\left(\frac{1}{\cancel{5}}\right)\left(1-\frac{1}{5^n}\right)}{\left(\frac{4}{\cancel{5}}\right)} = \frac{5^n - 1}{4 \cdot 5^n}.$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{5^n}\right) = \frac{a}{1-r} = \frac{\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)} = \frac{\left(\frac{1}{\cancel{5}}\right)}{\left(\frac{4}{\cancel{5}}\right)} = \frac{1}{4}$$