

Here is my solution:

If you made the counting stick have only 2 coloured rectangles, one blue and one yellow, you would end up with 3 rectangles. These are the:

- blue rectangle
- yellow rectangle
- Rectangle around the whole of the stick



If you added another coloured rectangle to this and made 3 coloured rectangles, you would have 6 rectangles in total. This would be:

- 3 coloured rectangles (3)



- Rectangle around the left two rectangles (+1)



- Rectangle around the right two rectangles (+1)



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- Rectangle around the whole of the stick (+1)



Already there is a pattern emerging:

Number of coloured rectangles	2	3
Number of rectangles	3	6

To be sure we must do it one more time. So, what happens when we add a fourth coloured rectangle?

There are:

4 coloured rectangles (4)



- The left two rectangles (+1)



- The middle two rectangles (+1)



- The right two rectangles (+1)



- The left 3 rectangles (+1)



- The right 3 rectangles (+1)



- The whole rectangle (+1)



Total = 10

That time we got 10 rectangles. Let's have a look at the table

Number of coloured rectangles	2	3	4
Number of rectangles	3	6	10

This data has now disproved the original theory that every time you add a coloured rectangle, there is 3 more rectangles. But another theory comes up. For the next one, we are going to see what happens when there is only 1 coloured rectangle.

There is:

- 1 coloured rectangle

Ok, so now our table looks like this:

Number of coloured rectangles (N)	1	2	3	4
Total Number of rectangles (R)	1	3	6	10

Jump: 2                      Jump: 3                      Jump: 4

This furthers our theory. This shows that as the number of coloured rectangles increases, the Jump of how many rectangles there are goes up by 1. For a start, from 1 to 2 coloured rectangles, the number of rectangles jumps by 2. Then for 2 to 3 coloured rectangles, the number of rectangles jumps by 3. This happens again when the number of rectangles jumps 4 to 10.

### **THE PROOF**

When adding a new coloured rectangle, the only new rectangles that it will make are ones that include that new coloured rectangle. Therefore, adding a new coloured rectangle (N) will create:

The new coloured rectangle itself (1)

A rectangle including the one immediately to its left (2)

A rectangle including the two immediately to its left (3)

All the way back to include all the rectangles (N)

Therefore, you always add on N new rectangles to the total.

Finally, we need to calculate the Nth term.

There are lots of videos online about how to do this, and here is one of them:

<https://www.youtube.com/watch?v=XaSyHze7AEs&t=77s>

After going through the steps, the Nth Term is

$$R = 0.5N \times (N+1)$$

With N being the number of coloured rectangles and R being the total number of rectangles.