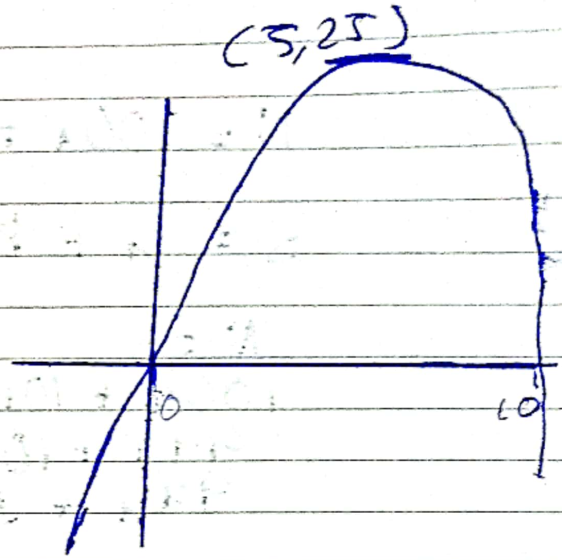
$0 \leq 10k - k^2 \leq 25$

$f(k) = 10k - k^2$
$f'(k) = 10 - 2k$



To find maximum:
$10 - 2k = 0$
$10 = 2k$
$k = 5$

$f(5) = 10(5) - 5^2$
$= 25$

So 25 is the maximum

The lowest value is 0
$f(0) = 10(0) - 0^2$
$= 0$

$\therefore \quad 0 \leq 10k - k^2 \leq 25$

$k(k-1)(k+3)$

k, $k-1$ and $k+1$ are all consecutive numbers. This means that at least one of them is a multiple of 2 and ~~at le~~ one of them is a multiple of

If k is a multiple of 3.
$k(k-1)(k+1)$ is a multiple of 3 as well

$\therefore$ This applies to $k+1$, and $k-1$.

$N = 100a + 10b + c$

$S = a + b^2 + c^3$

$N = S$

$100a + 10b + c = a + b^2 + c^3$

$99a + 10b - b^2 = c^3 - c$

$99a + b(10-b) = c(c+1)(c-1)$

$c(c+1)(c-1)$ is a multiple of 3.

$b(10-b)$ is between 0 and 25.

$c(c+1)(c-1)$ as $a \neq 0$.

This means that a multiple minus a number between 0 and 25 must equal to $c(c+1)(c-1)$

$c(c+1)(c-1)$ possibities:

120, 210, 336, 504, 720

Multiples of 99:

99, 198, 297, 396, 495, 594, 693, 792

$99a$ must be as close to $c(c+1)(c-1)$ but ~~bigge~~ smaller

To find $b(10-b)$:

$120 - 99 \times 1 = 21$

$a = 1$ $c(c+1)(c-1) = 120$ so $c = 5$

$b(10-b) = 21$

$b = 7$ or $b = 3$

$N = 100(1) + 10(7) + 5 = 175$

$N = 100(1) + 10(3) + 5 = 135$

Solutions

$210 - 2 \times 99 = 12$

$a = 2$, $c(c+1)(c-1) = 120 \therefore c = 6$

$b(10-b) = 12$

$b = 5 \pm \sqrt{13}$ non integer b solution ✗

$336 - 3 \times 99 = 39$ X

as $\cancel{b(10-b) > 25}$

$39 > 25$

$504 - 5 \times 99 = 9$

$a = 5$ $c(c+1)(c-1) = 504 \therefore c = 3$

$b(10-b) = 9$

$b = 9$ or $b = 1$

$N = 100(5) + 10(9) + 8 = 598$

$N = 100(5) + 10(1) + 8 = 518$

Solutions

$720 - 7 \times 99 = 27$ X

as $27 > 25$

$N = 5$

when $N = 175, 135, 598$ and 518