1. ~~Ta~~ $\phi(15)$

Total numbers less than 15 : 14 numbers

~~15 has the following factors : 1, 3, 5, 15 6, 9, 10~~

15 has the following numbers that are not co-prime (below 15):

1, 3, 5, 6, 9, 10

= 6 numbers

$\therefore \phi(15) = 14 - 6$

$= 8$

Hence, proved

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2. $\phi(p)$ where p is a prime number

By definition, if p is a ~~pr~~ prime number, then it will only have two factors: 1 and p

Hence, ~~every~~ no ~~nu~~ number less than p will share a factor with p. As a result, every number less than p ~~will will~~ will be co-prime to p. We know that there are ~~part~~ $(p-1)$ ~~numbers before less~~ positive integers less ~~th~~ than p.

$\therefore$ $\boxed{\phi(p) = p-1, \text{ where } p \text{ is a prime number}}$

Eg:

$\phi(n)$	(positive integers less than p which are co-prime)
$\phi(2) = 1$	(1)
$\phi(3) = 2$	(1, 2)
$\phi(5) = 4$	(1, 2, 3, 4)
$\phi(7) = 6$	(1, 2, 3, 4, 5, 6)
$\phi(11) = 10$	(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)

3. $2^n = 2, 4, 8, 16, 32, \ldots$ where $n = 1, 2, 3, 4, 5, \ldots$

2^n will only have powers of 2 as their factors. Since every 2^n will have 2 as a factor, all even ~~not~~ numbers less ~~th~~ than 2^n will not be ~~co~~ co-prime to 2^n.

Let us investigate ~~this~~ $\phi(2^n)$ for different values of 2^n

$\phi(2^1) = 1$	(1)
$\phi(2^2) = \cancel{2}\ 2$	(1, 3)
$\phi(2^3) = 4$	(1, 3, 5, 7)
$\phi(2^4) = 8$	(1, 3, 5, 7, 9, 11, 13, ~~14~~ 15)

From this, we can see $\phi(2^n) = \cancel{2^{n}}\ 2^{n-1}$, where $n = 1, 2, 3, 4, \ldots$

Similar to 2^n, only powers of 3 will ~~have~~ be factors of 3^n. Since 3 will be a factor of every ~~3^n~~ 3^n, all multiples of 3 less than 3^n will not be co-prime to 3^n.

Hence, to find $\phi(3^n)$, we need to subtract the ~~number~~ ~~of~~ multiples of 3 ~~from~~ from all numbers less than 3^n. This can be written as :-

$$\phi(3^n) = (3^n - 1) - \left(\frac{3^n - 3}{3}\right)$$

$$= 3^n - 1 - \frac{3^n}{3} + \frac{3}{3}$$

$$= 3^n\left(1 - \frac{1}{3}\right) \cancel{+} -1 + 1$$

$$= 3^n\left(1 - \frac{1}{3}\right)$$

$$= 3^n\left(\frac{2}{3}\right)$$

$$\boxed{\text{Hence, } \phi(3^n) = 3^n\left(\frac{2}{3}\right)}$$

$\phi(p^n)$, where p is ~~po~~ prime

~~If p is $= 2$, then $\phi(p^n) = 2^{n-1}$~~

~~If $p \neq 2$, then~~ We have to subtract all multiples of p less ~~than~~ than p^n from all numbers below p.
This can be written as:

$$\phi(p^n) = (p^n - 1) - \left(\frac{p^n}{p} - 1\right)$$

$$= p^n - 1 - \frac{p^n}{p} + 1$$

$$= p^n\left(1 - \frac{1}{p}\right)$$

$$= p^n\left(\frac{p-1}{p}\right)$$

$\boxed{\phi(p^n) = p^{n-1}(p-1)}$, If p is prime and not equal to 2