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Let $V_1$ and $V_2$ be the volumes of the smaller and larger spheres respectively, and $r_1$ and $r_2$ be the radiuses of the spheres.

As each sphere melts, its volume decreases at a rate directly proportional to its surface area.

$\therefore \quad \frac{dV_1}{dt} = kA_1$, where $k$ is a non-zero constant and $A_1$ is the surface area of small sphere

$\frac{dV_2}{dt} = kA_2$, where $A_2$ is surface area of large sphere,

By chain rule, $\frac{dV_1}{dt} = \frac{dV_1}{dr} \cdot \frac{dr_1}{dt} = A_1 \cdot \frac{dr_1}{dt}$

$\therefore \frac{dr_1}{dt} = k$, $r_1 = kt + c_1$, where $c_1$ is the initial value of the radius of small sphere.

Similarity, $r_2 = kt + c_2$, where $c_2$ is the initial value of the radius of the larger sphere.

Initially, when $t = 0$, $r_1 = 2R$ and $r_2 = 3R$.

$\therefore \quad c_1 = 2R$ and $c_2 = 3R$.

When Frosty the snowman is half its height,

$$r_1 + r_2 = \frac{1}{2}(5R)$$

$$2kt + 5R = 2.5R$$

$$t = \frac{-2.5R}{2k}$$

Value of $V_1$ at $t = \frac{-2.5R}{2k}$, $V_1 = \frac{4}{3}\pi (r_1)^3$

$$V_1 = \frac{4}{3}\pi \left(k\left(\frac{-2.5R}{2k}\right) + c_1\right)^3$$

$$V_1 = \frac{4}{3}\pi(-1.25R + 2R)^3 = \frac{4}{3}\pi(0.75R)^3$$

Similarity, $V_2 = \frac{4}{3}\pi(1.75R)^3$

Volume $V$ at $t = \frac{-2.5R}{2k}$ $= \frac{4}{3}\pi(0.75R)^3 + \frac{4}{3}\pi(1.75R)^3$

$$= \frac{185}{24}\pi R^3$$

Original volume $V_0$ at $t=0$, $\frac{4}{3}\pi(k(0) + c_1)^3 + \frac{4}{3}\pi(k(0) + c_2)^3$

$$= \frac{140}{3}\pi R^3$$

$$\frac{V}{V_0} = \left(\frac{185}{24}\pi R^3\right) \div \left(\frac{140}{3}\pi R^3\right) = \frac{37}{224}$$

$\therefore \quad V : V_0 = 37 : 224$ as required.

No.

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When Frosty the snowman is $\frac{1}{10}$ of his initial height,

$$r_1 + r_2 = \frac{1}{10}(5R)$$

$$2kt + 5R = 0.5R$$

$$t = \frac{-4.5R}{2k}$$

$$V_1 = \frac{4}{3}\pi\left(k\left(\frac{-4.5R}{2k}\right) + c_1\right)^3$$

$$= \frac{4}{3}\pi(-2.25R + 2R)^3 = \frac{4}{3}\pi(-0.25R)^3$$

$$V_2 = \frac{4}{3}\pi\left(k\left(\frac{-4.5R}{2k}\right) + c_2\right)^3$$

$$= \frac{4}{3}\pi(-2.25R + 3R)^3 = \frac{4}{3}\pi(0.75R)^3$$

$\because R > 0$, $-0.25R < 0$, $V_1 < 0$, the top snow ball has melted completely.

The ratio $= V_2 : V_0 = \frac{9}{16}\pi R^3 : \frac{140}{3}\pi R^3 = 27 : 2240$