

• Frosty is Melting!

$$\text{initial height} = 2 \times 2R + 2 \times 3R = 4R + 6R = 10R$$

$$\begin{aligned}\text{initial volume} &= \frac{4}{3} \times (2R)^3 \times \pi + \frac{4}{3} \times (3R)^3 \times \pi \\ &= \frac{32R^3\pi}{3} + \frac{108R^3\pi}{3} \\ &= \frac{140}{3} R^3\pi\end{aligned}$$

if we say  $x$  is the height of snow melted from each sphere,

$$\begin{aligned}\text{half height} &= 10R \div 2 = 5R = (4R - x) + (6R - x) \\ &= 10R - 2x\end{aligned}$$

$$2x = 5R, \quad x = \frac{5}{2}R$$

when Frosty is half his initial height, the radii of two snowballs are  $(4R - \frac{5}{2}R) \div 2 = \frac{3}{4}R$  and  $(6R - \frac{5}{2}R) \div 2 = \frac{7}{4}R$

$$\begin{aligned}\text{half volume} &= \frac{4}{3} \times (\frac{3}{4}R)^3 \times \pi + \frac{4}{3} \times (\frac{7}{4}R)^3 \times \pi \\ &= \frac{9}{16} R^3\pi + \frac{343}{48} R^3\pi \\ &= \frac{185}{24} R^3\pi\end{aligned}$$

$$\frac{185}{24} R^3\pi : \frac{140}{3} R^3\pi = 185 : 1120 = 37 : 224$$

• what is the ratio when Frosty is one-tenth of his initial height?

$$\text{height} = 10R \div 10 = R$$

$$\text{volume} = \frac{4}{3} \times (\frac{1}{2}R)^3 \times \pi = \frac{1}{6} R^3\pi$$

( $\because$  5R of snow melted from both spheres, but since the small sphere only has 4R of snow, the body sphere is the only one left)

$$\frac{1}{6} R^3\pi : \frac{140}{3} R^3\pi = 1 : 280$$