

$$\begin{aligned} \sqrt{x+5} &= x+3 && (1) \\ x+5 &= x^2+6x+9 && (2) \\ x^2+5x+4 &= 0 && (3) \\ (x+4)(x+1) &= 0 \\ x &= -4 \text{ or } x = -1 \end{aligned}$$

I think that -1 in the correct solution because in incorrect solution arises as a result of step (1)

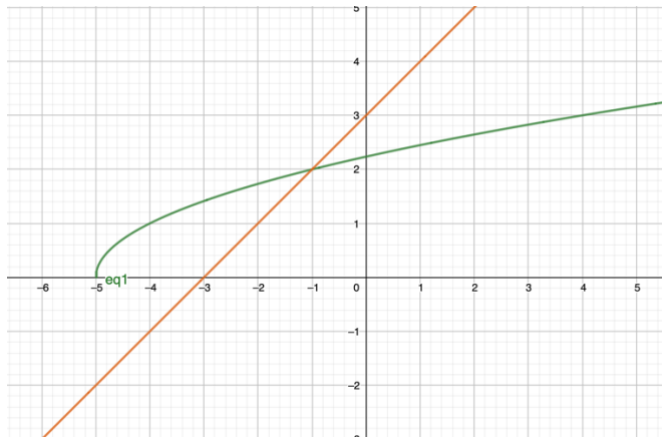
If we were to substitute -4 into the original equation, we would have the situation below:

$$\begin{aligned} \sqrt{-4+5} &= -4+3 \\ \sqrt{1} &\neq -1 \\ 1 &\neq -1 \end{aligned}$$

This is a problem because even though the both $(-1)^2$ and 1^2 have 1 as the solution, this does not mean that $-1 = 1$

The square root of 1 in the example is a type of **many-to-one function**, however the original question would be a case of **one-to-many**, which is not a function.

If we were to graph the original question using GeoGebra, then it would look something like this:



The green line is $\sqrt{x+5}$ and the orange line is $x+3$ and the point at which they intersect is -1. This graph proves that the original question has only one possible solution.

The -4 result arises because we will always get a **positive integer when we square numbers**, even if they are negative.

Both -4 and -1 become a solution when we square both sides of the equation like we did in **step (1)**, but if left in the original function form, then the question has **only one solution which is -1**.