The Difference of Odd Squares. | + Others.

- My conjecture about the differences of odd squares: The differences are all multiples of 8.

ⓐ $11^2 - 5^2 = 96$

ⓑ $5^2 - 3^2 = 16$

ⓒ $7^2 - 3^2 = 40$

* The Questions

My Observations

For the three questions, my first instinct was to add the two numbers being squared:- for ⓐ it was 16, ⓑ 8 and ⓒ 10. I then noticed that the sums were a factor of the answer of the differences. For ⓐ it's $16 \times 6 = 96$, ⓑ was $8 \times 2 = 16$, and c was $10 \times 4 = 40$. I noticed that in order for the factor to be the answer of the difference of the squares, it needed to be multiplied by the difference of the two numbers.

The Conjecture

Let two numbers a and b be odd:- (let a be bigger than b)

$$a^2 - b^2 = (a+b)(a-b)$$

The difference of the square of any two odd numbers is equal to the product of their sums and differences.

The Proof

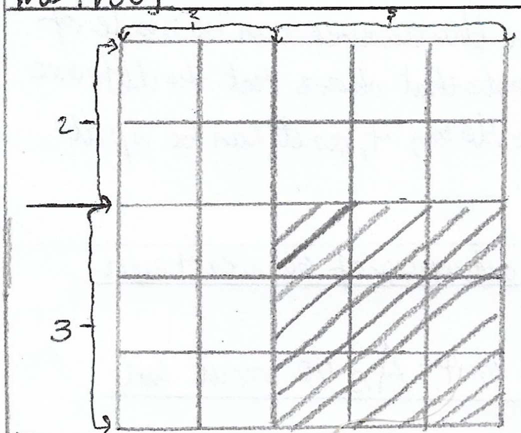
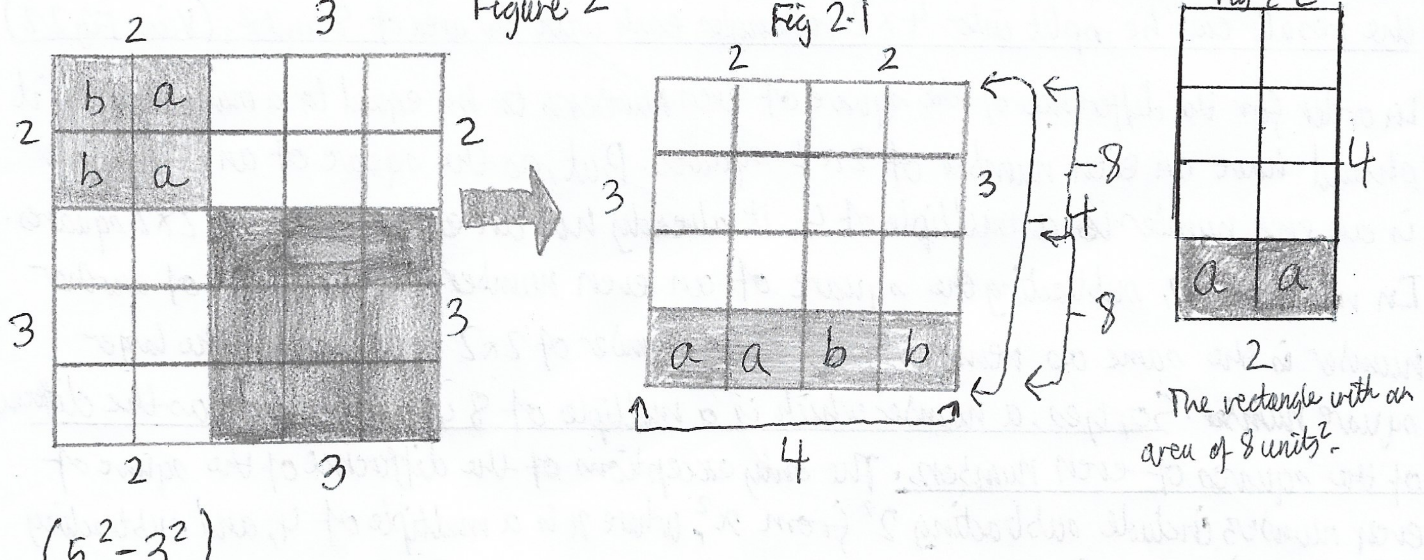


Fig. 1 * A 5×5 square ($5^2 - 3^2$)

For this proof, I've subtracted 9 (3^2) from a 5×5 square (5^2). It's noticeable that three shapes are formed as a result, two 3×2 rectangles and one 2×2 rectangle. This is an indication that to get the answer 16, 6 must be multiplied twice and 4 added to it. Furthermore, the light square which represent the difference of $5^2 - 3^2$ can be rearranged into a 4×4 square with an area of 16 units². 16 can be written as 8×2, an expression of two of its factors. And they can be obtained as (5+3) and (5-2) respectively; this is further demonstrated in figure 2.

Figure 2



$(5^2 - 3^2)$

The rectangle with an area of 8 units².

While the answer can be represented as a 4×4 square, it is also possible to represent it as a shape made out of 2 ~~4~~×2 rectangles, which geometrically represent (8)×(2) which also is (5+3)×(5-3)

Difference of the Squares of Even Numbers:

$10^2-8^2=36$

$8^2-4^2=48$

$12^2-6^2=108$

$4^2-2^2=12$

These results are always divisible by 4.

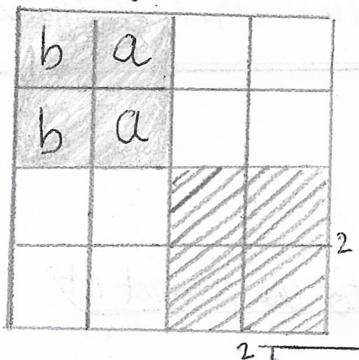
Observations

- The result's ~~always~~ a multiple of 6.
- The difference of the numbers being squared is a factor of the answer. Dividing the answer by the difference of the numbers gives us the sum of the two numbers being squared.

The rule that $a^2-b^2=(a+b)(a-b)$ works even if a and b are even. Furthermore, $(a+b)$ and $(a-b)$ will always be divisible by 4 if a and b are even.

Proof:- Let $a=4$ and $b=2$.

Fig 3·1



4^2-2^2

Fig. 3

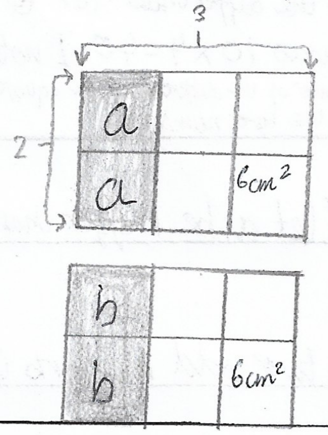
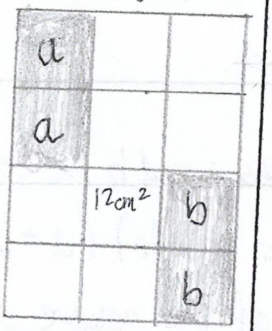


Fig 3·3



This visual proof demonstrates that the result of 4^2-2^2 can be split into two 3×2 rectangles each with an area of 6cm². Combined, they give an area of 12 cm². This is one where the answer can be made of 3 2×2 squares. This is one of the examples that shows that the difference of the square of even numbers is divisible by 4, as it can be split into 2×2 squares.

In odd numbers:-
$a^2-b^2=$ a multiple of 8.

In even numbers:-
$a^2-b^2=$ a multiple of 4.

Q. Can a number which is a multiple of 8 be written as the difference of squares of even numbers.

- The difference of the square of even numbers is a multiple of 4. As the result can be split into 2×2 squares. (View Fig 3·3).
- On the other hand, the difference of the square of odd numbers is a multiple of 8. As the result can be split into 4×2 rectangles each with an area of 8 units². (View Fig 2·2).

In order for the difference of the square of even numbers to be equal to a multiple of 8, it should have an even number of 2×2 squares. But, as the square of an even number is an even number is a multiple of 4, it already has an even number of 2×2 squares. In most cases, subtracting the square of an even number by the square of another number is the same as removing an even number of 2×2 squares from the larger square number. So, yes, a number which is a multiple of 8 can be written as the difference of the squares of even numbers. The only exceptions of the difference of the square of even numbers include subtracting 2^2 from x^2, where x is a multiple of 4, and subtracting a^2 $((b-2)^2)$ from b^2. As in both these instances an odd number of 2×2 squares is removed from the larger square.

Formula for finding out whether the square of two even numbers is a multiple of 8.

a. $x^2 - 2^2 = y$

If x is a multiple of 4 and not a multiple of 8, y is not a multiple of 8.

b. $a^2 - b^2 = c$ (where a and b are even).

$$\frac{a^2}{4} - \frac{b^2}{4}$$

$\Downarrow$ $\Downarrow$

A B

Assuming $a > b$:-

If A − B = An odd number:-

c is not a multiple of 8

$C = 4(A-B)$

C will only be a multiple of 8 if A − B is an even number.

Difference between the squares of odd and even numbers:

$9^2 - 6^2 = 45$ $9^2 - 2^2 = 77$ $9^2 - 8^2 = 17$

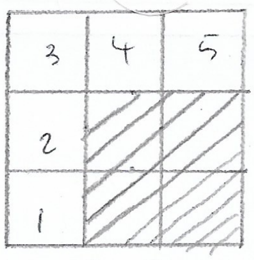
$7^2 - 4^2 = 33$ $11^2 - 4^2 = 105$ $11^2 - 2^2 = 117$

Firstly while the rule that $a^2 - b^2 = (a+b)(a-b)$ still applies to the above displayed problems. Some really interesting connections are noted:-

① $9^2 - 8^2 = 17$

As $17 = 9 + 8$. It can be theorised that the difference of two consecutive numbers squared equals to the sum of the two numbers itself. As demonstrated in the below solution of $3^2 - 2^2$:-

* However, sums which are a multiple of 10 are doubled.



$3^2 - 2^2 = 5$ Fig. 4.

Proof:-

$n^2 - (n-1)^2 = n^2 - (n-1)^2$

$\Rightarrow n^2 - (n^2 - 2n + 1)$

$\Rightarrow n^2 - n^2 + 2n - 1$

$\Rightarrow n^2 - (n-1)^2 = 2n - 1$

$2 \times (3) - 1$

$\Rightarrow 6 - 1$

$\Rightarrow 5$

Knowing that if n is odd, then n − 1 should be even. The above logic can be applied for similar difference of squared numbers:

Eg. $11^2 - 2^2 = n^2 - ((n-9)^2)$

$\Rightarrow (11+2)(11-2)$ $\Rightarrow n^2 - (n^2 - 18n + 81)$

$\Rightarrow 117$ $\Rightarrow n^2 - n^2 + 18n \pm 81$

$\Rightarrow 117 = 18n + 81 = \cancel{18 \times 2 + 81 = 117}$

$117 + 81 = 18n$

$19\cancel{8}\ \cancel{36} = 18n$

$11\ \cancel{8} = n$

$11^2 - 2^2 = 117$

Another quicker way to recreate the formula would be to multiply the distance between the two numbers by 2 and keep the result positive. And then, after multiplying n to the algorithm $(-2(a)n)$, subtract the square of the number from the previous step. $(-2(a)n - (a^2))$. Once created, substitute n with the largest squared number as shown in the example.

* Exception being if one of the digits is 1.

Q: What numbers can be written as the difference of two squares.

- Nearly all positive and negative numbers can be written as the difference of two squares. Almost all even numbers can be written as the difference of two squared numbers.

Q: What numbers can't be written as the difference of two squares.

0, 1 and 2 can't be expressed in this manner. The only negative number which can't be expressed as a difference of 2 squared numbers is −2.

Conclusion

- The difference of two squares $(a^2-b^2)=(a+b)(a-b)$.
- The difference of the squares of odd numbers always equals to a multiple of 8.
- The difference of the square of even numbers always equals to a multiple of 4.
- When the difference is between the square of an even or odd number with its counterpart - The results vary and can include prime numbers.
- Using the same idea of ~~seperat~~ subtracting bigger numbers from smaller numbers to obtain negative numbers. The same idea could apply to squared numbers to produce any negative number but −2.