DIFFERENCE OF ODD SQUARES

$$11^2 - 7^2 = 72$$
$$19^2 - 13^2 = 192$$

- Initial thoughts were that all are multiple of 4 and coincidentally a multiple of 8. But 72 and 192 are also multiples of 8. So conjecture is that difference of odd squares is a multiple of 8.

- Let the 2 odd numbers be $(2a+1)$ and $(2b+1)$.

$$(2a+1)^2 - (2b+1)^2 = [(2a+1) + (2b+1)][(2a+1) - (2b+1)]$$
$$= [2(a+b+1)][2(a-b)]$$
$$= 4(a+b-1)(a-b)$$

- Odd, even cases

b

	ODD $(2n+1)$	EVEN $(2n)$
a: ODD $(2m+1)$	$= 4(2m+1+2n+1-1)(2m+\cancel{1}-2n-\cancel{1})$ $= 8(2m+1+2n)(m-n)$	$= 4(2m+\cancel{1}+2n-\cancel{1})(2m+1-2n)$ $= 8(m+n)(2m+1-2n)$
a: EVEN $(2m)$	$= 4(2m+2n+\cancel{1}-\cancel{1})(2m-2n-1)$ $= 8(m+n)(2m-2n-1)$	$= 4(2m+2n-1)(2m-2n)$ $= 8(2m+2n-1)(m-n)$

★ Taking parity of m and n in cases can also give multiple of 16 in some cases

As seen, in every case, the difference of odd squares is indeed a multiple of 8.

OTHER QUESTIONS

- Let the even numbers be $(2a)$ and $(2b)$

Then, difference of squares is $(2a)^2 - (2b)^2$

$$(2a)^2 - (2b)^2 = (2a+2b)(2a-2b)$$
$$= 2(a+b) \cdot 2(a-b)$$
$$= 4(a+b)(a-b) \quad \text{OR} \quad = 4(a^2-b^2)$$

And this would be a multiple of 4

- For this to be a multiple of 8, $(a+b)(a-b)$ must be even
$(a^2 - b^2)$ must be even

And this is only possible if both $a \& b$ are even or both $a \& b$ are odd

- If, $(2a+1)$ and $(2b)$ are odd and even, difference of squares

$$(2a+1)^2 - (2b)^2 = (2a+1+2b)(2a+1-2b) \longrightarrow \text{odd, no multiple of 2}$$

- So, we've seen that difference of two odd squares is always a multiple of 8.
Difference of two (multiples of 4) squares ~~are~~ is also a multiple of 8
And difference of squares of two ~~not~~ even numbers not divisible by 4, also gives a multiple of 8