

## NRICH Maths Challenge – Difference of odd squares

### Rohan - Wilson's School

$$\begin{aligned} 3^2 - 1^2 &= 8 \\ 5^2 - 1^2 &= 24 \\ 7^2 - 1^2 &= 48 \\ 9^2 - 1^2 &= 80 \\ 11^2 - 1^2 &= 120 \\ 5^2 - 3^2 &= 16 \\ 7^2 - 5^2 &= 24 \end{aligned}$$

I tested various outcomes for the difference of squares of odd numbers. I noticed that the outcomes are all divisible by 4.

#### How to prove this:

I first noticed that if you took two consecutive odd numbers, then the difference between the squares can be calculated using the following formula:

$$4a - 4 \quad (\text{where } a \text{ is the larger odd number})$$

Example:  $5^2 - 3^2 = 25 - 9 = \underline{16}$

Using the formula  $4a - 4$ , we can work out:

$$(5 \times 4) - 4 = \underline{16}$$

However, this only worked where the numbers were consecutive. I then started to look at outcomes with non-consecutive numbers and found the following formula:

$$(a + b) \times (a - b) \quad (\text{where } a \text{ is the larger odd number})$$

Example:  $7^2 - 3^2 = 49 - 9 = \underline{40}$

In this case,  $a = 7$  and  $b = 3$

$$(7 + 3) \times (7 - 3)$$

$$10 \times 4 = \underline{40}$$

**Other questions to think about:**

- a) The formula  $(a + b) \times (a - b)$  also works for outcomes where we take the difference of even numbers squared:

Example:  $8^2 - 2^2 = 64 - 4 = \underline{60}$

In this case,  $a = 8$  and  $b = 2$

$$(8 + 2) \times (8 - 2)$$

$$10 \times 6 = \underline{60}$$

- b) I tested various even numbers using the formula  $(a + b) \times (a - b)$  and found that there are many instances where the answer is a multiple of 8.

Examples:

$$6^2 - 2^2 \dots\dots (6 + 2) \times (6 - 2) \dots\dots 8 \times 4 = \mathbf{32}$$

$$8^2 - 4^2 \dots\dots (8 + 4) \times (8 - 4) \dots\dots 12 \times 4 = \mathbf{48}$$

$$10^2 - 6^2 \dots\dots (10 + 6) \times (10 - 6) \dots\dots 16 \times 4 = \mathbf{64}$$

I created a table and highlighted yellow boxes where the values, from the formula  $a^2 - b^2$ , gave a multiple of 8 (both  $a$  and  $b$  are even numbers). A repeating pattern was identified.

|    | b     | 2     | 4     | 6     | 8     | 10    | 12    | 14    | 16    | 18    | 20    | 22    | 24    |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|    | $b^2$ | 4     | 16    | 36    | 64    | 100   | 144   | 196   | 256   | 324   | 400   | 484   | 576   |
| a  | $a^2$ |       |       |       |       |       |       |       |       |       |       |       |       |
| 2  | 4     | 0     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     |
| 4  | 16    | 12    | 0     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     |
| 6  | 36    | 32    | 20    | 0     | -     | -     | -     | -     | -     | -     | -     | -     | -     |
| 8  | 64    | 60    | 48    | 28    | 0     | -     | -     | -     | -     | -     | -     | -     | -     |
| 10 | 100   | 96    | 84    | 64    | 36    | 0     | -     | -     | -     | -     | -     | -     | -     |
| 12 | 144   | 140   | 128   | 108   | 80    | 44    | 0     | -     | -     | -     | -     | -     | -     |
| 14 | 196   | 192   | 180   | 160   | 132   | 96    | 52    | 0     | -     | -     | -     | -     | -     |
| 16 | 256   | 252   | 240   | 220   | 192   | 156   | 112   | 60    | 0     | -     | -     | -     | -     |
| 18 | 324   | 320   | 308   | 288   | 260   | 224   | 180   | 128   | 68    | 0     | -     | -     | -     |
| 20 | 400   | 396   | 384   | 364   | 336   | 300   | 256   | 204   | 144   | 76    | 0     | -     | -     |
| 22 | 484   | 480   | 468   | 448   | 420   | 384   | 340   | 288   | 228   | 160   | 84    | 0     | -     |
| 24 | 576   | 572   | 560   | 540   | 512   | 476   | 432   | 380   | 320   | 252   | 176   | 92    | 0     |
| 26 | 676   | 672   | 660   | 640   | 612   | 576   | 532   | 480   | 420   | 352   | 276   | 192   | 100   |
| 28 | 784   | 780   | 768   | 748   | 720   | 684   | 640   | 588   | 528   | 460   | 384   | 300   | 208   |
| 30 | 900   | 896   | 884   | 864   | 836   | 800   | 756   | 704   | 644   | 576   | 500   | 416   | 324   |
| 32 | 1,024 | 1,020 | 1,008 | 988   | 960   | 924   | 880   | 828   | 768   | 700   | 624   | 540   | 448   |
| 34 | 1,156 | 1,152 | 1,140 | 1,120 | 1,092 | 1,056 | 1,012 | 960   | 900   | 832   | 756   | 672   | 580   |
| 36 | 1,296 | 1,292 | 1,280 | 1,260 | 1,232 | 1,196 | 1,152 | 1,100 | 1,040 | 972   | 896   | 812   | 720   |
| 38 | 1,444 | 1,440 | 1,428 | 1,408 | 1,380 | 1,344 | 1,300 | 1,248 | 1,188 | 1,120 | 1,044 | 960   | 868   |
| 40 | 1,600 | 1,596 | 1,584 | 1,564 | 1,536 | 1,500 | 1,456 | 1,404 | 1,344 | 1,276 | 1,200 | 1,116 | 1,024 |

- c) I created a table of values using the formula  $a^2 - b^2$  where  $a$  is an odd number and  $b$  is an even number. Several of the values were multiples of 3 and these have been highlighted in yellow below. A repeating pattern can be identified.

|    |                | b              | 2     | 4     | 6     | 8     | 10    | 12    | 14    | 16    | 18    | 20    | 22    | 24  |
|----|----------------|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|
|    |                | b <sup>2</sup> | 4     | 16    | 36    | 64    | 100   | 144   | 196   | 256   | 324   | 400   | 484   | 576 |
| a  | a <sup>2</sup> |                |       |       |       |       |       |       |       |       |       |       |       |     |
| 1  | 1              |                | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -   |
| 3  | 9              |                | 5     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -   |
| 5  | 25             |                | 21    | 9     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -   |
| 7  | 49             |                | 45    | 33    | 13    | -     | -     | -     | -     | -     | -     | -     | -     | -   |
| 9  | 81             |                | 77    | 65    | 45    | 17    | -     | -     | -     | -     | -     | -     | -     | -   |
| 11 | 121            |                | 117   | 105   | 85    | 57    | 21    | -     | -     | -     | -     | -     | -     | -   |
| 13 | 169            |                | 165   | 153   | 133   | 105   | 69    | 25    | -     | -     | -     | -     | -     | -   |
| 15 | 225            |                | 221   | 209   | 189   | 161   | 125   | 81    | 29    | -     | -     | -     | -     | -   |
| 17 | 289            |                | 285   | 273   | 253   | 225   | 189   | 145   | 93    | 33    | -     | -     | -     | -   |
| 19 | 361            |                | 357   | 345   | 325   | 297   | 261   | 217   | 165   | 105   | 37    | -     | -     | -   |
| 21 | 441            |                | 437   | 425   | 405   | 377   | 341   | 297   | 245   | 185   | 117   | 41    | -     | -   |
| 23 | 529            |                | 525   | 513   | 493   | 465   | 429   | 385   | 333   | 273   | 205   | 129   | 45    | -   |
| 25 | 625            |                | 621   | 609   | 589   | 561   | 525   | 481   | 429   | 369   | 301   | 225   | 141   | 49  |
| 27 | 729            |                | 725   | 713   | 693   | 665   | 629   | 585   | 533   | 473   | 405   | 329   | 245   | 153 |
| 29 | 841            |                | 837   | 825   | 805   | 777   | 741   | 697   | 645   | 585   | 517   | 441   | 357   | 265 |
| 31 | 961            |                | 957   | 945   | 925   | 897   | 861   | 817   | 765   | 705   | 637   | 561   | 477   | 385 |
| 33 | 1,089          |                | 1,085 | 1,073 | 1,053 | 1,025 | 989   | 945   | 893   | 833   | 765   | 689   | 605   | 513 |
| 35 | 1,225          |                | 1,221 | 1,209 | 1,189 | 1,161 | 1,125 | 1,081 | 1,029 | 969   | 901   | 825   | 741   | 649 |
| 37 | 1,369          |                | 1,365 | 1,353 | 1,333 | 1,305 | 1,269 | 1,225 | 1,173 | 1,113 | 1,045 | 969   | 885   | 793 |
| 39 | 1,521          |                | 1,517 | 1,505 | 1,485 | 1,457 | 1,421 | 1,377 | 1,325 | 1,265 | 1,197 | 1,121 | 1,037 | 945 |

I also prepared a table of values using the formula  $a^2 - b^2$  where  $a$  is an even number and  $b$  is an odd number. Multiples of 3 were identified and highlighted in yellow. A repeating pattern was also identified.

|    |                | b              | 1     | 3     | 5     | 7     | 9     | 11    | 13    | 15    | 17    | 19    | 21    | 23    |
|----|----------------|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|    |                | b <sup>2</sup> | 1     | 9     | 25    | 49    | 81    | 121   | 169   | 225   | 289   | 361   | 441   | 529   |
| a  | a <sup>2</sup> |                |       |       |       |       |       |       |       |       |       |       |       |       |
| 2  | 4              |                | 3     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     |
| 4  | 16             |                | 15    | 7     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     |
| 6  | 36             |                | 35    | 27    | 11    | -     | -     | -     | -     | -     | -     | -     | -     | -     |
| 8  | 64             |                | 63    | 55    | 39    | 15    | -     | -     | -     | -     | -     | -     | -     | -     |
| 10 | 100            |                | 99    | 91    | 75    | 51    | 19    | -     | -     | -     | -     | -     | -     | -     |
| 12 | 144            |                | 143   | 135   | 119   | 95    | 63    | 23    | -     | -     | -     | -     | -     | -     |
| 14 | 196            |                | 195   | 187   | 171   | 147   | 115   | 75    | 27    | -     | -     | -     | -     | -     |
| 16 | 256            |                | 255   | 247   | 231   | 207   | 175   | 135   | 87    | 31    | -     | -     | -     | -     |
| 18 | 324            |                | 323   | 315   | 299   | 275   | 243   | 203   | 155   | 99    | 35    | -     | -     | -     |
| 20 | 400            |                | 399   | 391   | 375   | 351   | 319   | 279   | 231   | 175   | 111   | 39    | -     | -     |
| 22 | 484            |                | 483   | 475   | 459   | 435   | 403   | 363   | 315   | 259   | 195   | 123   | 43    | -     |
| 24 | 576            |                | 575   | 567   | 551   | 527   | 495   | 455   | 407   | 351   | 287   | 215   | 135   | 47    |
| 26 | 676            |                | 675   | 667   | 651   | 627   | 595   | 555   | 507   | 451   | 387   | 315   | 235   | 147   |
| 28 | 784            |                | 783   | 775   | 759   | 735   | 703   | 663   | 615   | 559   | 495   | 423   | 343   | 255   |
| 30 | 900            |                | 899   | 891   | 875   | 851   | 819   | 779   | 731   | 675   | 611   | 539   | 459   | 371   |
| 32 | 1,024          |                | 1,023 | 1,015 | 999   | 975   | 943   | 903   | 855   | 799   | 735   | 663   | 583   | 495   |
| 34 | 1,156          |                | 1,155 | 1,147 | 1,131 | 1,107 | 1,075 | 1,035 | 987   | 931   | 867   | 795   | 715   | 627   |
| 36 | 1,296          |                | 1,295 | 1,287 | 1,271 | 1,247 | 1,215 | 1,175 | 1,127 | 1,071 | 1,007 | 935   | 855   | 767   |
| 38 | 1,444          |                | 1,443 | 1,435 | 1,419 | 1,395 | 1,363 | 1,323 | 1,275 | 1,219 | 1,155 | 1,083 | 1,003 | 915   |
| 40 | 1,600          |                | 1,599 | 1,591 | 1,575 | 1,551 | 1,519 | 1,479 | 1,431 | 1,375 | 1,311 | 1,239 | 1,159 | 1,071 |