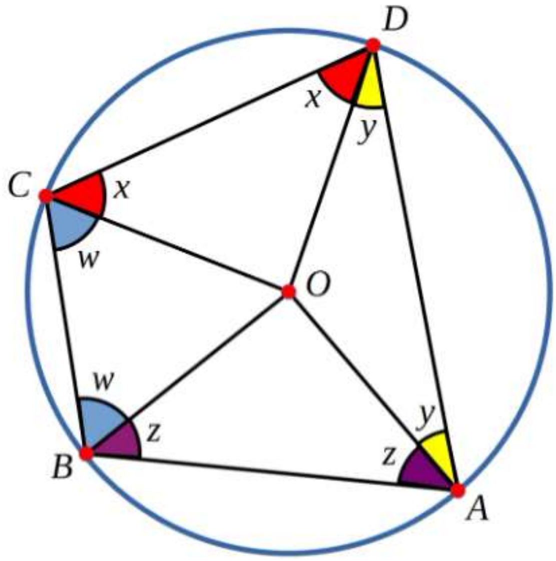
NRICH MATHS CHALLENGE - Cyclic Quadrilaterals Proof
Rohan - Wilson's School



My proof below shows that opposite angles of a cyclic quadrilateral add to 180°. Based on the above diagram that we were provided, this means that w + z + x + y = 180°.

The diagram shows that the quadrilateral has been divided into four different isosceles triangles (OAB, OBC, OCD & OAD). It also has letters allocated for the quadrilateral angles. I have therefore created labels below for the angles around the centre of the circle. My labels are based on the mathematical rule that the three angles in a triangle sum to 180°:

1. Triangle OAB
   Angle AOB = 180° - 2z

2. Triangle OBC
   Angle BOC = 180° - 2w

3. Triangle OCD
   Angle COD = 180° - 2x

4. Triangle OAD
   Angle AOD = 180° - 2y

We also know the mathematical rule that angles around a point sum to 360°. Based on this rule, we can create the equation below:

Angle AOB + Angle BOC + Angle COD + Angle AOD = 360°

180° - 2z + 180° - 2w + 180° - 2x + 180° + 2y = 360°

If we re-arrange this equation, we get:

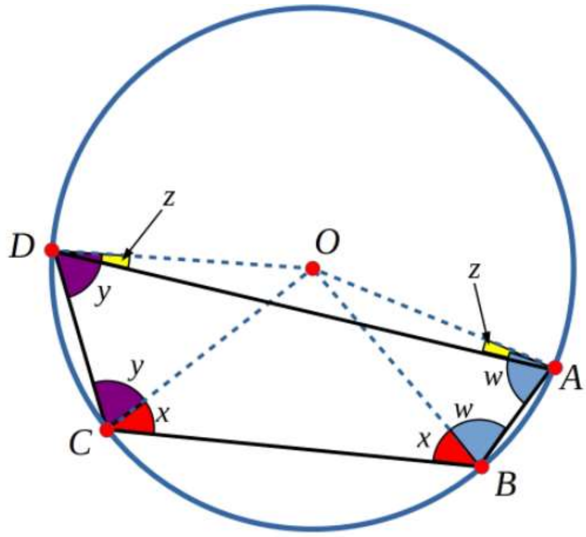
180° + 180° + 180° + 180° - 360° = 2z + 2w + 2x + 2y

2x + 2y + 2w + 2z = 360°

Therefore, w + x + y + z = 180°

This proves that opposite angles of a cyclic quadrilateral sum to 180°.

Extension



The diagram shows a cyclic quadrilateral when the centre of the circle is outside the quadrilateral. Once again, the quadrilateral has been divided into isosceles triangles from the centre point of the circle. The triangles are OCD, OBC & OAB. Based on the mathematical rule that the three angles in a triangle sum to 180°:

1. Triangle OCD
   Angle COD = 180° - 2y

2. Triangle OBC
   Angle BOC = 180° - 2x

3. Triangle OAB
   Angle AOB = 180° - 2w

Since the quadrilateral is outside the centre of the circle, there is also triangle OAD.

4. Triangle OAD
   Angle AOD = 180° - 2z

Angle AOD = Angle COD + Angle BOC + Angle AOB

180° - 2z = 180° - 2y + 180° - 2x + 180° - 2w

If we re-arrange this equation, we get:

2w + 2x + 2y – 2z = 180° + 180° + 180° - 180°

2w + 2x + 2y – 2z = 360°

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Therefore, $w + x + y - z = 180°$

In this case, angle z (Angle ODA) is subtracted as it sits outside the quadrilateral.

This also proves that opposite angles of a cyclic quadrilateral sum to 180°, even where the centre of the circle sits outside the quadrilateral.