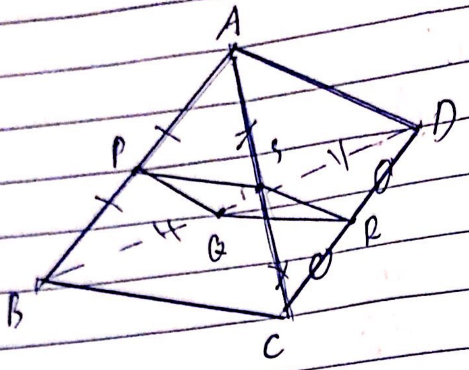
# Tetra Slice




In $\Delta ABD$, P and Q are the midpoints of AB and BD, respectively.
$\Rightarrow$ PQ is the midline of $\Delta ABD$
$\Rightarrow$ $PQ \parallel AD$ and $PQ = \frac{1}{2} AD$ (which could also be deduced by using similar triangles) (1)
In $\Delta ACD$, S and R are the midpoints of AC and CD, respectively,
$\Rightarrow$ SR is the midline of $\Delta ACD$
$\Rightarrow$ $SR \parallel AD$ and $SR = \frac{1}{2} AD$ (2)
From deductions (1) and (2) $\Rightarrow$ $PQ \parallel SR$ and $PQ = SR$
$\Rightarrow$ PQRS is a parallelogram. $\blacksquare$

Extension: If ABCD is a regular tetrahedron $\Rightarrow$ $\Delta ABC = \Delta ADC$
$= \Delta BCD = \Delta ABD$ (property of a regular tetrahedron)
$\Rightarrow$ $BS = SD$ (as they are ~~[illegible]~~ perpendicular bisectors of $\Delta ABC$ and $\Delta ADC$, respectively)
And Q is the midpoint of BD in ~~[illegible]~~ $\Delta BSD$ isoceles at S
$\Rightarrow$ SQ is the perpendicular bisector of $\Delta BSD$ (3)
Likewise, ~~[illegible]~~ PR is the perpendicular bisector of $\Delta PCD$
Consider $\Delta BSD$ and $\Delta$ ~~PCD~~ CPD:
BD = DC (as $\Delta BDC$ is an equilateral triangle)
BS = CP (both are perpendicular bisectors of $\Delta ABC$)
SD = PD (SD and PD are perpendicular bisectors of congruent triangles ADC and ABD, respectively)
$\Rightarrow$ $\Delta BSD = \Delta CPD$ (s-s-s) (4)

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And PR is the perpendicular bisector of $\Delta PCD$ isoceles at P (R is midpoint of DC) (5)

From deductions (3), (4), (5) => PR = QS

Yet PQRS is also a parallelogram

=> PQRS is a rectangle (6)

As previously concluded, $\begin{cases} PS = \frac{1}{2}BC \\ PQ = \frac{1}{2}AD \end{cases}$

ABCD is a regular tetrahedron => BC = AD

=> PS = PQ (7)

From deductions (6) and (7) => PQRS is a square