a) Let the elves be A1, B1 and C1 and their besties A2, B2 and C2 respectively.

Without loss of generality, assume A1 sits at the head of the table.

(we can do this as all arrangements can be rotated to this, so it also prevents double counting)

We have 5 elf choices right of A1, 4 elf choices right of the next elf, then 3, and so on, giving $5! = 120$ possible arrangements.

To have all A1 next to their bestie, A2 can go to their left or right (2 options).

Continuing round in the same direction, we can have any of the 4 remaining elves next, but next we're forced to have their bestie.

The next elf can be either of the 2 left, but the final elf is again forced.

This gives $2 \times 4 \times 1 \times 2 \times 1 = 16$ arrangements where all besties are together.

So the probability is $\frac{16}{120} = \frac{2}{15}$, as we aimed to show.

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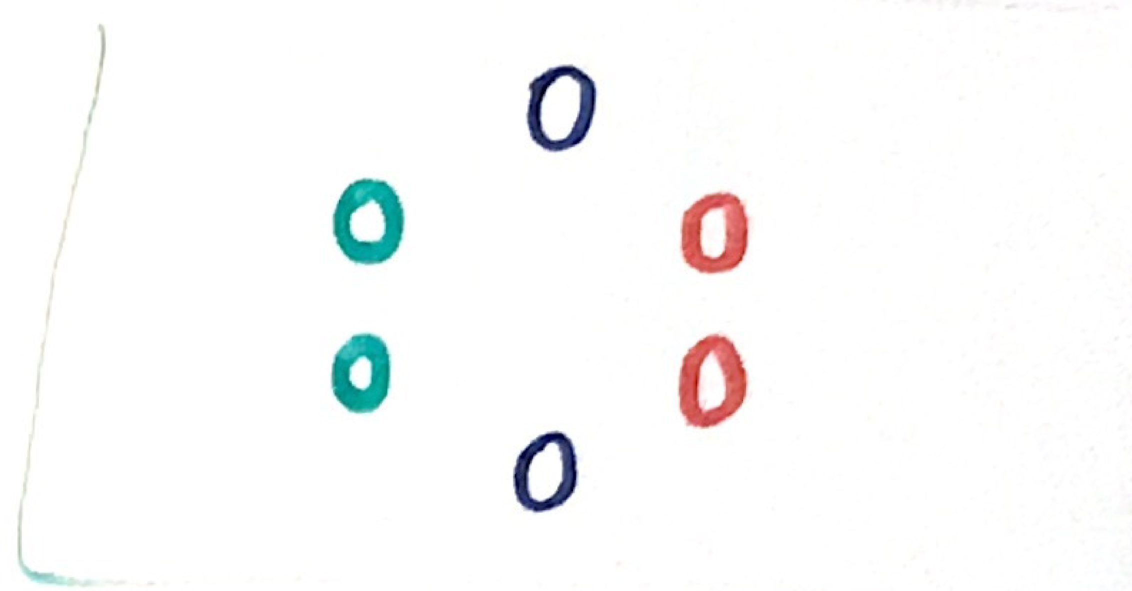
b) In order for 2 pairs to be together, we need an arrangement like the one to the left.



We chose one of 3 pairs to be blue, one of the remaining 2 to be green, and red is forced.

WLOG and to prevent double counting, suppose elf 1 of the blue pair sits at the top of the table. We then have 2 arrangements for each of the red and green pairs.

This gives $3 \times 2 \times 2 \times 2 = 24$ working arrangements.

Thus the probability is $\frac{24}{120} = \frac{1}{5}$.

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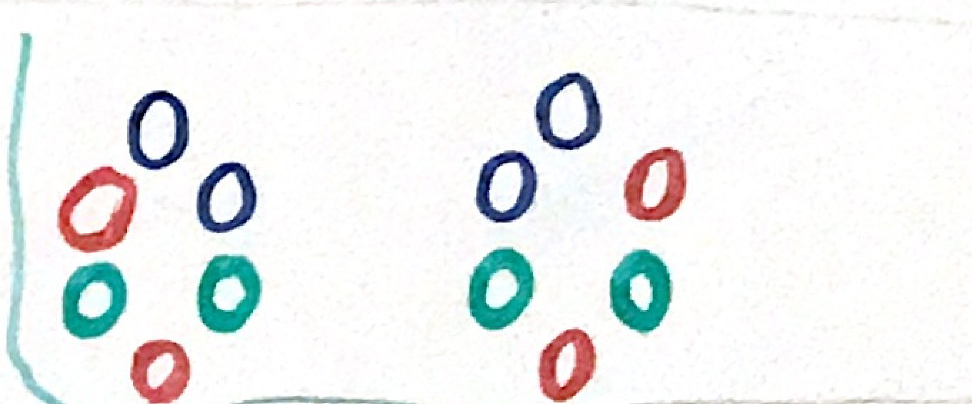
c) The arrangements where no pair sits together is 120 - (all together) - (2 together) - (1 together). So let's consider the case that 1 pair is together. To the right we see the only 2 possible arrangements.



WLOG and to prevent double counting, partner 1 sits at the top of the table.

We assign pairs to colours as before, in $3 \times 2 = 6$ ways. The blue pair have fixed order to prevent double counting, but red and green each have 2 ways to move around.

This makes $6 \times 2 \times 2^2 = 48$ arrangements where we only have one pair.

Thus we have $120 - 16 - 24 - 48 = 32$ arrangements with no pairs.

Thus the probability that no besties are together is $\frac{32}{120} = \frac{4}{15}$ □