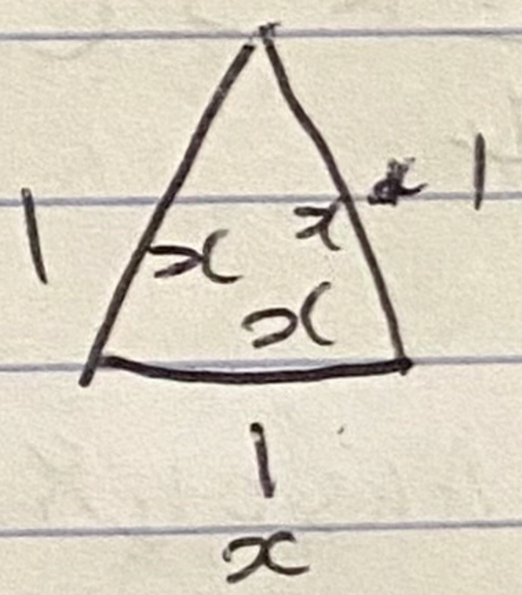
The Koch Snowflake - Qs

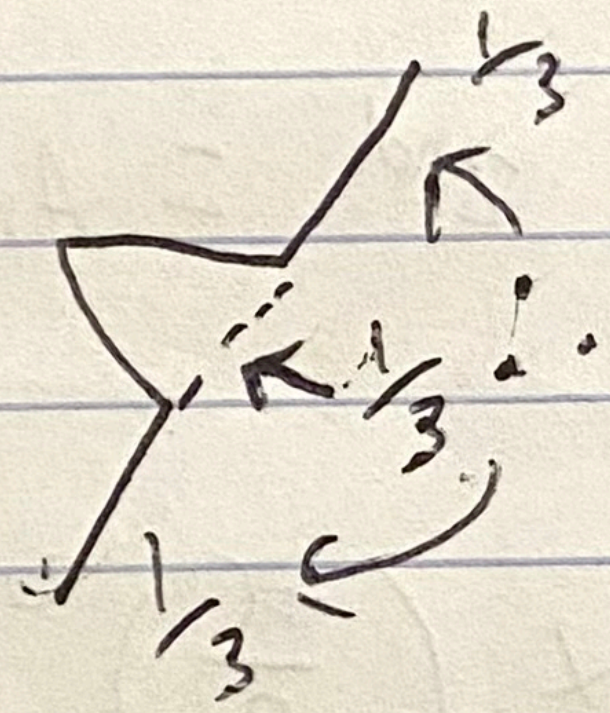
1) First Iteration - 3 edges
Second It... - 12 edges
Third It... - 48 edges

2) formula for edges → $\boxed{E_n = 3 \times 4^{n-1}}$

$E_0 = 3 \times 4^0$ ~~(1st Sequence = E_0)~~
$E_1 = 3 \times 4^1$ ~~(Use Sequence n - 1 for n)~~
$E_2 = 3 \times 4^2$

3) $\boxed{1/3}$





4) Length of edges → $\boxed{L_n = \cancel{1\times}\left(\tfrac{1}{3}\cancel{x}\right)^{n-1}}$

$1^{st} = 1$ $E_0 = 1$ ~~$E_n = 1 \times (1/3)^n$~~
$2^{nd} = 1/3$ $E_1 = 1/3$ ~~E_0 = 1st Sequence~~
$3^{rd} = 1/9$ $E_2 = 1/9$

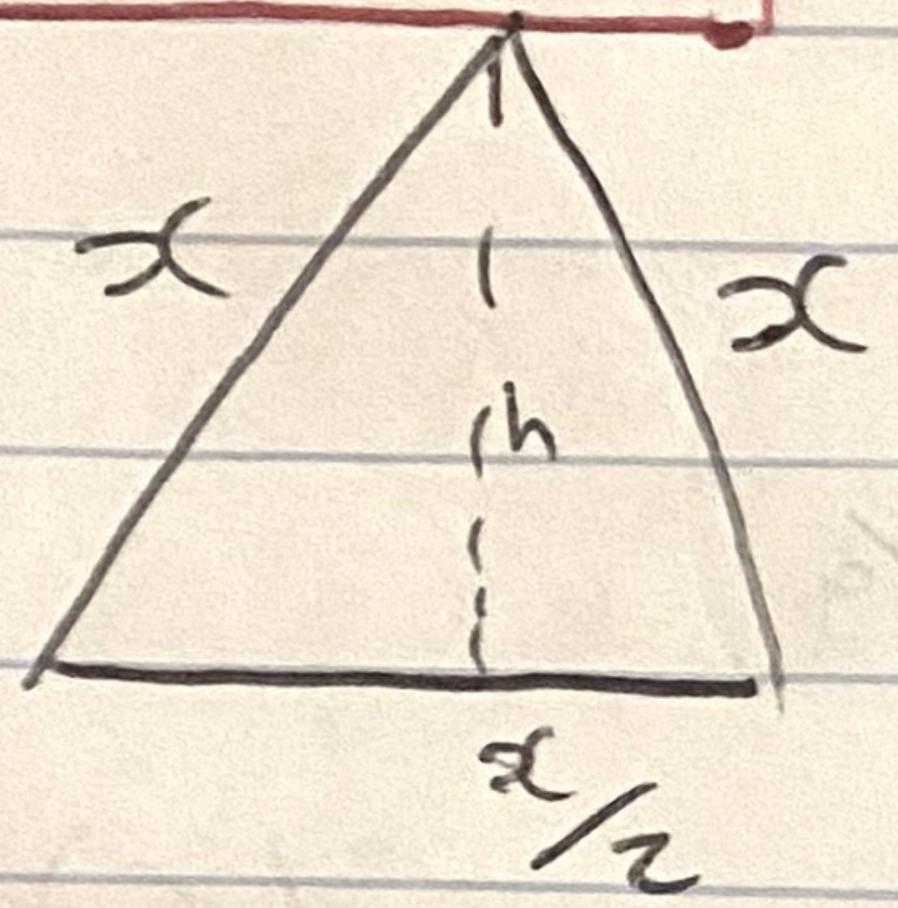
5) Perimeter → $\boxed{P_n = \left(\tfrac{1}{3}\cancel{x}\right)^{n-1} \times 4^{n-1} \times 3}$ → No of sides of the triangle

No of edges
No of these on one side

$A_1 = 3$
$A_2 = 4$
$A_3 = \cancel{8}\ (\tfrac{1}{3})^2 \times 4^2 \times 3 = 5.11$

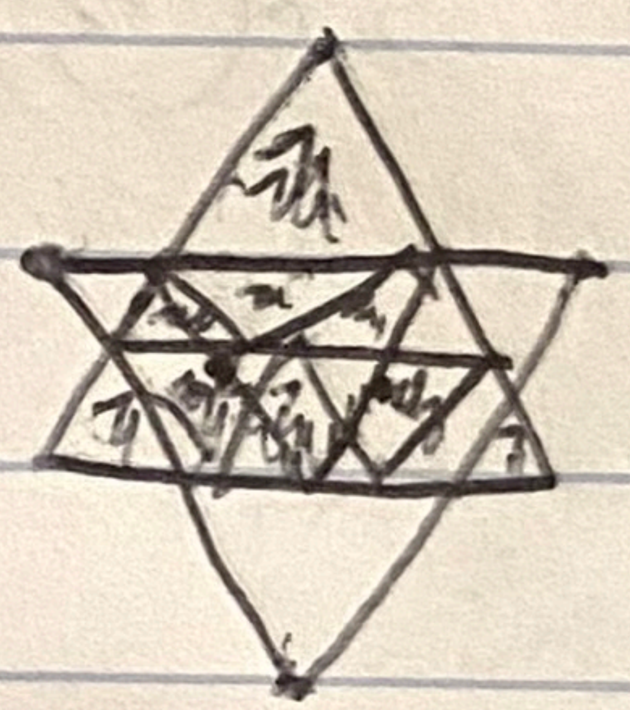
Area of Koch Snowflake

1) $\boxed{\frac{\sqrt{3}x^2}{36}}$ $x = 1$ $\boxed{\frac{\sqrt{3}}{36}}$



$$h = \sqrt{x^2 - \frac{x^2}{4}} = \frac{3x^2}{4} = \frac{\sqrt{3}x}{2}$$

$$\frac{\sqrt{3}x}{2} \times \frac{x}{2} = \frac{\sqrt{3}x^2}{4} \to A$$



$$\frac{A}{9} \to \frac{\sqrt{3}x^2}{4} \times \frac{1}{9} = \frac{\sqrt{3}x^2}{36} \to \text{Each mini Triangle}$$

$$\frac{\sqrt{3}x^2}{36} \times 3 = \frac{\sqrt{3}x^2}{12}$$

2) $\boxed{\frac{\sqrt{3}x^2}{12}}$ or If $x = 1$ $\boxed{\frac{\sqrt{3}}{12}}$

3) $\boxed{0.06415 \text{ (4.S.F)}}$

Adding Previous Area as it remains

$$\text{Area} = \overset{1}{\frac{\sqrt{3}}{4}x^2} + \overset{2}{3 \times \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2}$$

No. of New Triangles

Formula for area of the new triangles

length of each edge decreases by a factor of a $\frac{1}{3}$ each time

$$\ldots + \underbrace{\overset{3}{12 \times \frac{\sqrt{3}}{4}\left(\frac{x}{9}\right)^2}}_{\text{Extra Area added}} \qquad 12 \times \frac{\sqrt{3}}{4}\left(\frac{1}{9}\right)^2 = 0.06415 \text{ (4.S.F)}$$

$x = 1$

4) $$\text{Infinite Sum} = \overset{1}{\frac{\sqrt{3}}{4}x^2} + \overset{2}{3 \times \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2} +$$

$$\overset{3}{\underline{3 \times 4} \times \frac{\sqrt{3}}{4}\left(\frac{x}{9}\right)^2} + \overset{4}{3 \times 4^2 \times \frac{\sqrt{3}}{4}\left(\frac{x}{27}\right)^2} \ldots$$

Instead of writing 12. As it would increase by a factor of 12 with every iteration.

To Simplify this, you can factorise each term by $\frac{\sqrt{3}x^2}{4}$ as every term contains these numbers.

$$\text{Simplified} = \frac{\sqrt{3}x^2}{4}\left(\overset{1}{1} + \overset{2}{3\left(\frac{1}{3}\right)^2} + \overset{3}{3 \times 4\left(\frac{1}{9}\right)^2} + \overset{4}{3 \times 4^2\left(\frac{1}{27}\right)^2} \ldots\right)$$

Simplified further: It can be simplified even more if you write each iteration with one power instead of two.

$$\rightarrow \frac{\sqrt{3}x^2}{4}\left(1 + 3\left(\frac{1}{9}\right) + 3\times4\left(\frac{1}{9^2}\right) + 3\times4^2\left(\frac{1}{9^3}\right)\ldots\right)$$

However, the powers on the 4s are one less than the 9s, so we need to increase it. We can do this by multiplying the sequence by 4 and dividing the factor by 4.

$$\rightarrow \frac{\sqrt{3}x^2}{4}\underline{\div 4}\overset{\times 4}{\left(4 + 3\times\underline{4}\left(\frac{1}{9}\right) + 3\times\underline{4^2}\left(\frac{1}{9^2}\right)\ldots\right)}$$

can be written as

$$3\times\left(\frac{4^2}{9^2}\right) = 3\times\left(\frac{4}{9}\right)$$

If you combined it, you get:

$$\rightarrow \frac{\sqrt{3}x^2}{16}\left(4 + 3\left(\frac{4}{9}\right) + 3\left(\frac{4}{9}\right)^2 + 3\left(\frac{4}{9}\right)^3\ldots\right)$$

~~Finally~~ Lastly, you can factor out a 3:

$$\rightarrow \frac{3\sqrt{3}x^2}{16}\left(\overset{1}{\frac{4}{3}} + \overset{2}{\left(\frac{4}{9}\right)} + \overset{3}{\left(\frac{4}{9}\right)^2} + \overset{4}{\left(\frac{4}{9}\right)^3} + \overset{5}{\left(\frac{4}{9}\right)^4}\ldots\right)$$

~~As~~ Length - As the number of sides increase, the length increases by a factor of 4/3 of the perimeter, so as the No of sides approaches ∞, the length will also approach ∞ (infinite length)

Area - $\left(\frac{4}{9}\right)^2 = 0.198$ $\left(\frac{4}{9}\right)^3 = 0.0878$

This tells us that as the number of sides increase, the added area of each triangle ~~gets~~ decreases. Eventually, the area will reach a limit $\therefore$ the area is finite.