<u>Tri-angled trig</u>, 3sbur

15/04/23

given: $\theta + \phi + \psi = \frac{\pi}{2}$ (1)

: $\sin(\theta + \phi + \psi) = \sin(\frac{\pi}{2})$

$\sin((\theta+\phi)+\psi) = 1$

$\cancel{\sin\theta\cos\phi + \sin\phi\cos\theta}$

$\sin(\theta+\phi)\cos\psi + \sin\psi\cos(\theta+\phi) = 1$

$(\sin\theta\cos\phi + \cos\theta\sin\phi)\cos\psi + \sin\psi(\cos\theta\cos\phi - \sin\theta\sin\phi) = 1$

look for easier method — above method too long so I took a different approach:

alternatively, re-arranging (1): $\theta + \phi = \frac{\pi}{2} - \psi$

$\Rightarrow \sin(\theta+\phi) = \sin(\frac{\pi}{2} - \psi)$ (sin of both sides)

$\Rightarrow \sin\theta\cos\phi + \sin\phi\cos\theta = \cos\psi$ (double angle formulae)

$\Rightarrow \sin^2\phi\cos^2\theta + \sin^2\theta\cos^2\phi + 2\sin\theta\sin\phi\cos\theta\cos\phi = \cos^2\psi$ (squaring both sides)

$\Rightarrow \sin^2\phi + \sin^2\theta - \sin^2\phi\sin^2\theta - \sin^2\theta\sin^2\phi$ (using $\cos^2 x = 1 - \sin^2 x$

$\Rightarrow + 2\sin\theta\sin\phi\cos\theta\cos\phi = \cos^2\psi$

$\Rightarrow \sin^2\phi + \sin^2\theta + \sin^2\psi - 2\sin^2\phi\sin^2\theta + 2\sin\theta\sin\phi\cos\theta\cos\phi = 1$

$\Rightarrow \sin^2\phi + \sin^2\theta + \sin^2\psi + 2\sin\theta\sin\phi(\cos\theta\cos\phi - \sin\theta\sin\phi) = 1$

$\Rightarrow \sin^2\phi + \sin^2\theta + \sin^2\psi + 2\sin\theta\sin\phi(\cos(\theta+\phi)) = 1$

since: $\cos(\theta+\phi) = \cos(\frac{\pi}{2} - \psi) = \sin\psi$ (from $\theta+\phi+\psi = \frac{\pi}{2}$)

we have:

$\sin^2\phi + \sin^2\theta + \sin^2\psi + 2\sin\theta\sin\phi\sin\psi = 1$

tri-angled trig

cont...

note :

$$\sin\frac{\pi}{5} = \sin\left(\frac{\pi}{10}+\frac{\pi}{10}\right) \qquad \text{if } \theta = \phi = \frac{\pi}{5}$$

$$\psi = \frac{\pi}{2} - \frac{2\pi}{5} = \frac{\pi}{10}$$

$$= 2\sin\frac{\pi}{10}\cos\frac{\pi}{10}$$

then subbing values into equation.

$$\sin^2\frac{\pi}{5} + \sin^2\frac{\pi}{5} + \sin^2\frac{\pi}{10} + 2\sin\frac{\pi}{5}\sin\frac{\pi}{5}\sin\frac{\pi}{10} = 1$$

$$\Rightarrow \quad 2\sin^2\frac{\pi}{5} + 2\sin^2\frac{\pi}{5}\sin\frac{\pi}{10} + \sin^2\frac{\pi}{10} = 1$$

$$\Rightarrow \quad 8\sin^2\frac{\pi}{10}\cos^2\frac{\pi}{10} + 8\sin^3\frac{\pi}{10}\cos^2\frac{\pi}{10} + \sin^2\frac{\pi}{10} = 1$$

$$\Rightarrow \quad \sin^2\frac{\pi}{10}\left(8\cos^2\frac{\pi}{10} + 8\sin\frac{\pi}{10}\cos^2\frac{\pi}{10} + 1\right) = 1$$

$$\cos^2\frac{\pi}{10} = 1 - \sin^2\frac{\pi}{10}$$

so ~~sin~~ we have :

$$\sin^2\frac{\pi}{10}\left(8 - 8\sin^2\frac{\pi}{10} + 8\sin\frac{\pi}{10} - 8\sin^3\frac{\pi}{10} + 1\right) = 1$$

let $\sin\frac{\pi}{10} = x$ above becomes

$$x^2(9 - 8x^2 + 8x - 8x^3) = 1$$

$$9x^2 - 8x^4 + 8x^3 - 8x^5 = 1$$

$$\Rightarrow \quad -8x^5 - 8x^4 + 8x^3 + 9x^2 - 1 = 0$$

$$\Rightarrow \quad 8x^5 + 8x^4 - 8x^3 - 9x^2 + 1 = 0$$

substituting $x=1$ gives 0 and also substituting $x=-1$ gives 0

↳ (x^2-1) is a factor

$$\Rightarrow \quad (x^2-1)(8x^3+8x^2-1) = 0$$

(by 'inspection')

$$x^2 - 1 = \sin^2\frac{\pi}{10} - 1 \neq 0$$

$\therefore$ $\sin\frac{\pi}{10}$ satisfies equation $8x^3+8x^2-1=0$