$$\theta + \phi + \psi = \frac{\pi}{2} \Rightarrow \sin(\theta + \phi + \psi) = 1$$

$$\sin((\theta + \phi) + \psi) = 1$$

$$\sin(\theta + \phi)\cos\psi + \sin\psi \cos(\theta + \phi) = 1$$

(1) $\sin\theta \cos\phi \cos\psi + \sin\phi \cos\theta \cos\psi + \sin\psi \cos\theta \cos\phi - \sin\theta \sin\phi \sin\psi = 1$

Note: $\cos\phi \cos\psi = \cos(\phi + \psi) + \sin\phi \sin\psi = \sin\theta + \sin\phi \sin\psi$ where $\theta + \phi + \psi = \frac{\pi}{2}$.

Similarly, $\cos\theta \cos\psi = \sin\phi + \sin\theta \sin\psi$ and $\cos\theta \cos\phi = \sin\psi + \sin\theta \sin\phi$

Thus, we can simplify (1);

$$\sin^2\theta + \sin\theta \sin\phi \sin\psi + \sin^2\phi + \sin\theta \sin\phi \sin\psi + \sin^2\psi = 1$$

$$\Rightarrow \sin^2\theta + \sin^2\phi + \sin^2\psi + 2\sin\theta \sin\phi \sin\psi = 1$$

If $\theta = \phi = \frac{\pi}{5}$, then $\frac{2\pi}{5} + \psi = \frac{\pi}{2} \Rightarrow 2\psi = \frac{\pi}{5} = \theta = \phi$

So,

$$\sin^2(2\psi) + \sin^2(2\psi) + \sin^2(\psi) + 2\sin^2(2\psi)\sin(\psi) = 1$$

$$2\sin^2(2\psi) + \sin^2(\psi) + 2\sin^2(2\psi)\sin(\psi) = 1$$

$$2(4\sin^2\psi \cos^2\psi) + \sin^2\psi + 2(4\sin^2\psi \cos^2\psi)\sin\psi = 1$$

$$8\sin^2\psi(1 - \sin^2\psi) + \sin^2\psi + 8\sin^3\psi(1 - \sin^2\psi) = 1$$

$$8\sin^2\psi - 8\sin^4\psi + \sin^2\psi + 8\sin^3\psi - 8\sin^5\psi = 1$$

$$8\sin^5\psi + 8\sin^4\psi - 8\sin^3\psi - 9\sin^2\psi + 1 = 0$$

Let $\sin(\psi) = \sin(\frac{\pi}{10}) = s$,

$$8s^5 + 8s^4 - 8s^3 - 9s^2 + 1 = 0$$

$$(s-1)(8s^4 + 16s^3 + 8s^2 - s - 1) = 0$$

$$(s-1)(s+1)(8s^3+8s^2-1)=0$$

$$\Rightarrow \quad 8s^3+8s^2-1=0 \quad \Rightarrow \quad 8\left(\sin\left(\tfrac{\pi}{10}\right)\right)^3+8\left(\sin\left(\tfrac{\pi}{10}\right)\right)^2-1=0$$

$\therefore$ $x=\sin\left(\frac{\pi}{10}\right)$ is a solution to the equation $8x^3+8x^2-1=0$