$(2x+1)^2 = 4x^2+4x+1$

$\frac{4x^2+4x+1}{8} = \frac{4x^2}{8}+\frac{4x}{8}+\frac{1}{8} = \frac{x^2}{2}+\frac{x}{2}+\frac{1}{8}$

- If x is even then x and x^2 are both even and therefore divisible by two which means that the division of the terms leaves a remainder of 1.
- If x is odd then x and x^2 have a sum that is even (because the sum of two odd numbers is even) and therefore divisible by two which means that the division of the terms leaves a remainder of 1.

$(2x)^2 = 4x^2$

$\frac{4x^2}{8}$

- If x is even, the product of two multiples of four is divisible by eight, so the remainder is 0.
- If x is odd, the product of 4 and x^2 is a multiple of 4, but is not a multiple of 8, since x^2 is odd, so the remainder is 4, because it is halfway between two multiples of eight.

8n+7, where n is a positive integer, can not be expressed as a sum of three squares, because 8n+7 gives a remainder of 7 in mod 8 and as the previous proofs show that square numbers divided by eight leave remainders of 0, 1 or 4.
In mod 8:
0+0+0≠7
0+0+1≠7
0+1+1≠7
1+1+1≠7
0+0+4≠7
0+4+4≠7
4+4+4≠7
0+1+4≠7
1+1+4≠7
1+4+4≠7
So
$8n + 7 \neq x^2 + y^2 + z^2$