Square Remainders - Zimei Xu (Gilbert)

Part 1:

odd numbers can be expressed as $2n+1$ or $2n-1$

$(2n+1)^2 = 4n^2+4n+1$

$4n^2+4n+1 = 4n(n+1)+1$

if n = odd

$4 \times$ odd $\times$ (odd + 1) + 1 = $4 \times$ odd $\times$ (multiple of 2) + 1

if n = even

$4 \times$ even $\times$ (even + 1) + 1 = $4 \times$ (multiple of 2) $\times$ (odd) + 1

if $n = 0$

$4 \times 0 \times (0+1)+1 = 1$ $\qquad$ $1 \div 8$ leaves remeinder of 8

$\because$ $4 \times$ (multiple of 2) = multiple of 8 & anything $\times$ multiple of 8 = multiple of 8

$\therefore$ $(2n+1)^2$ = (multiple of 8) + 1

which will leave a remainder of 1, when divided by 8

even numbers are $2n$

$(2n)^2 = 4n^2$

if n = odd

$4 \times (\text{odd})^2 = 4 \times$ odd , odd $= 2x+1$

$= 4 \times (2x+1)$

$= 8x + 4$ $\qquad$ $8x$ is divisible by 8

4 leaves a remainder of 4

if n = even or 0

$4 \times (\text{even})^2 = 4 \times$ even , even $= 2x$

$= 4 \times 2x$

$= 8x$ $\qquad$ $8x$ is divisible by 8

Therefore no remeinder

$4 \times (0)^2 = 0 \qquad 0 \div 8 = 0 \ldots 0$

$\therefore$ even numbers square divided by 8 will leave either 4 or 0

Part 2 :

$8n + 7$ cannot be expressed as a sum of three squares

$8n+7$ divided by 8 will leave a remeinder of 7

every odd number squre leaves remeinder 1 when divided by 8

every even number squre leaves remeinder 4 or 0 when divided by 8.

The sum of 3 square can never have a remainder of 7,

Prove : 3 odds ; $1+1+1$ = remeinder of 3

2odds and even : $1+1+4$ = remeinder of 6

or

$1+1+0$ = remeinder of 2

2even and odd : $1+4+4 = 8+1$ = remeinder of 1

$1+4+0$ = remerinder of 5

$1+0+0$ = remeinder of 1

3 even : 4 or 0 can never make 7.

$4+4+4 = 8+4$: remeinder of 4

$4+4+0 = 8$: remeinder of 0

$4+0+0$ = remeinder of 4

$0+0+0$ = remeinder of 0

$\therefore$ Because out of all the combinations of odd and even squres, there will never be an expression where the remeinder of the sum of three squsre when divided by 8 is 7, we can conclude that $8n+7$ cannot be expressed as the sum of three squres.