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Proper Factor

(1.) Show that $3^2 \times 5^3$ has exactly 10 proper factors.

Answer: For $3^2 \times 5^3$, it has $(2+1) \times (3+1) = 3 \times 4$
$= 12$ factors

A proper factor of an integer N is a positive integer, not 1 or N, that divides N.
So 12 factors minus 2 equals 10 factors
All of them are on the excel table

For $3^m \times 5^n$, it has $(m+1) \times (n+1)$ total number of factors

$\therefore (m+1) \times (n+1) - 2 =$ total number of proper factors.

$(m+1) \times (n+1) - 2 = 10$
$(m+1) \times (n+1) = 12$

$$(m+1) \times (n+1) = \begin{cases} 2 \times 6 \\ 6 \times 2 \\ 3 \times 4 \\ 4 \times 3 \end{cases}$$

Possible Combos

m+1	n+1	m	n	$3^m \times 5^n$	EXCEL TABLE
2	6	1	5	$3^1 \times 5^5$	Table (2)
6	2	5	1	$3^5 \times 5^1$	Table (3)
3	4	2	3	$3^2 \times 5^3$	Table (4) = same as table (1)
4	3	3	2	$3^3 \times 5^2$	Table (5)

(2.) Let this number which has 426 proper factors be N
$N = 2^m \times 3^n \times 5^p \times 7^q$

This has a total factor equals to $426 + 2 = 428$

$\therefore (m+1) \times (n+1) \times (p+1) \times (q+1) = 428$
$= 2 \times 2 \times 107$

$$\begin{array}{r|l} 2 & 428 \\ 2 & 214 \\ & 107 \end{array}$$

N must be the smallest positive integer that has 428 factors.
Assign 107 to $m+1$
① $\therefore m = 106$ $\quad 2^m$ is 2^{106}

② Assign 2 to $n+1 \Rightarrow 3^1$
$\therefore n = 1$
③ Assign 2 to $p+1 \Rightarrow 5^1$
$\therefore p = 1$

$\therefore N = 2^m \times 3^n \times 5^p \times 7^q$
$\therefore N = 2^{106} \times 3^1 \times 5^1 \times 1$

$N = 1.2 \times 10^{33}$
Solutions include excel file.