1. This can be done through brute force. By trying all pairs of powers we get 3,5,9,15,45,25,75,225,125, 375. I achieved this by forming a table as seen below with ascending powers of 3 running along the top and ascending powers of 5 running down. To find the amount of "co-ordinates" on our graph we can simply do our length $\cdot$ width but we have to remember to subtract 2 since a proper factor cannot be 1 or N.

$3^0, 5^0$	$3^1, 5^0$	$3^2, 5^0$
$3^0, 5^1$	$3^1, 5^1$	$3^2, 5^1$
$3^0, 5^2$	$3^1, 5^2$	$3^2, 5^2$
$3^0, 5^3$	$3^1, 5^3$	$3^2, 5^3$

This makes the 2nd part rather easier to visualise as we extend both axis of our graph to an arbitrary pair of values of m, n. The total possible co-ordinates is still given by the same formula - our length would be $m + 1$ and our width would be $n + 1$ giving the total possible amount of pairs as $(m + 1)(n + 1)$ but as before, we have to subtract the top left and bottom right pairs as they would multiply to give us 1 and N which does not satisfy the conditions of a proper factor. $\Rightarrow$ the amount of proper factors $z = (m + 1)(n + 1) - 2$.

$3^0 5^0$	$3^1 5^0$	$3^2 5^0$	...	...
$3^0 5^1$	$3^1 5^1$	$3^2 5^1$	...	...
$3^0 5^2$	$3^1 5^2$	$3^2 5^2$	...	...
$3^0 5^3$	$3^1 5^3$	$3^2 5^3$	...	...
...	...	...	...	$3^m 5^n$

The question is considering then considers when $z = 10$ so substituting this value into our newfound expression gives us $(m + 1)(n + 1) = 12$. We can spot a symmetry in m, n in this expression or in other words, if the pair (a, b) is a solution then (b, a) must also be a solution which can act as a short cut but is not essential to the problem.

Since $m, n \in \mathbb{N}$, we can consider all the possible pairs of whole numbers that multiply to give 12. This could be tackled using prime factorisation for larger N but since 12 is a very small case, I just wrote all the possible pairs of numbers down : $(1, 12), (2, 6), (3, 4)$ and naturally we could swap the order of each set of pairs to get another valid pair so keep that in mind when we do our calculations. $\Rightarrow (m + 1) = 1, 2, 3$ and $(n + 1) = 12, 6, 4$ which leaves us with 3 pairs of m, n but recalling our symmetry, we can swap the order and still get a valid result, so we multiply our result by 2 to achieve a total possible 6 integers with exactly 10 proper factors. :)