3. If $n-1$ is divisible by 3, show that n^3-1 is divisible by 9

If $n-1$ is divisible one, Than we can set a value $3x$, which is divisible by 3 so $n-1=x$, $n=3x+1$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$n^3-1=n^3-1^3$$

$$\therefore n^3-1^3=(n-1)(n^2+n+1^2)$$

$$=3x[(3x+1)^2+(3x+1)+1]$$

$$=3x(9x^2+6x+1+3x+1+1)$$

$$=3x(9x^2+9x+3)$$

$$=3x\cdot 3(3x^2+3x+1)$$

$$=9x(3x^2+3x+1)$$

If $3x$ is divisible by three than $3\cdot 3x=9x$ $3x$ a number that is divisible by 3 equal to a number that is divisible by 9

$\therefore n^3-1^3$ is divisible by 9

$\therefore n^3-1$ is divisible by 9

Divisible Factorisation

Harris Chan

1. Factorise n^5-n^3, and show that it is divisible by 24

$n^5-n^3 = n^2(n^3-n)$ $\quad 24 = 2^3 \times 3$

$2^2(2^3-2)$ $\quad 4 \times (8-2) = 4 \times 6 = 24$

$3^2(3^3-3)$ $\quad 3^3-3 = 24$ $\quad 9 \times 24 = 2^3 \times 3^3$

$4^2(4^2-4)$ $\quad 4^3-4 = 60$ $\quad 16 \times 60 = 2^6 \times 3 \times 5$

$5^2(5^3-5)$ $\quad 5^3-5 = 120$ $\quad 25 \times 120 = 5^3 \times 2^3 \times 3$

⋮

$even^2 = even$ $\quad n^3-n$ always divisible by 3

$odd^2 = odd$ $\quad odd-odd = even$ $\quad even-even = even$ $\quad even$ is divisible by 2

If n is an odd number, n^3-n always divisible by 24

- If n is an even number than n^2 always at least include 2^2 as it's factor and n^3-n at least include 2 as it's factor ($2^2 \times 2 = 8$) and n^3-n always divisible by 3 so $8 \times 3 = 24$

2. Prove that $2^{2n}-1$ is divisible by 3

$2^2-1 = 3$ $\quad 3 \div 3 = 1$ even index. are divisible by 3

$2^3-1 = 7$ $\quad 7$ is not divisible by 3

$2^4-1 = 15$ $\quad 15 \div 3 = 5$

$2^5-1 = 31$ $\quad 31$ is not divisible by 3 odd index aren't divisible by 3

⋮

- even x even = even $\quad$ even x odd = even $\quad 2$ is an even number

So $2n$ is even $\therefore 2^{even}-1$ can be divisible by 3 $\therefore 2^{2n}-1$ is divisible by 3