Preliminary work :

We will start by examining the equation $x^2 - y^2 = n$, where x, y and n are positive integers.

We can factor the left-hand side to obtain $(x - y)(x + y) = n$.

Now suppose that $x - y = a$ and $x + y = b$, which means $n = ab$ and $a < b$. We will now solve for x and y in terms of a and b:

$$\begin{cases} x - y = a & (1) \\ x + y = b & (2) \end{cases}$$

By adding equations (1) and (2), we obtain $2x = a + b$ or $x = \frac{a+b}{2}$.
By subtracting (1) from (2), we obtain $2y = b - a$ or $y = \frac{b-a}{2}$.

For x and y to be integers, $a + b$ and $a - b$ must be even, which means a and b must have the same parity (they must be either both odd or both even).

We conclude that every integer n can be written in the form $x^2 - y^2$, where:

i. $x = \frac{a+b}{2}$ and $y = \frac{b-a}{2}$
ii. $ab = n$
iii. $a < b$
iv. a and b have the same parity

We are now ready to tackle the questions.

Solution to #1:

For each case, we find integers a and b that satisfy the conditions and use that to find x and y:

n	a	b	$x = \dfrac{a+b}{2}$	$y = \dfrac{b-a}{2}$	$n = x^2 - y^2$
3	1	3	2	1	$3 = 2^2 - 1^2$
5	1	5	3	2	$5 = 3^2 - 2^2$
8	2	4	3	1	$8 = 3^2 - 1^2$
12	2	6	4	2	$12 = 4^2 - 2^2$
16	2	8	5	3	$16 = 5^2 - 3^2$

Note for the next few questions: If n can be expressed as a difference of squares, then there exist positive integers a and b that satisfy the conditions above. This means that if we cannot find positive integers a and b that satisfy the conditions, then n cannot be expressed as a difference of squares.

Solution to #2:

Note that any odd number n can be written in the form $2k + 1$, where k is an integer.
We see that $a = 1$ and $b = 2k + 1$ satisfy the conditions. Hence, any odd number can be expressed as a difference of two squares. ■

Solution to #3:

Here, $n = 4k$. We see that $a = 2$ and $b = 2k$ satisfy the conditions. Hence, any number of the form $4k$, where k is a non-negative integer, can be expressed as a difference of two squares. ■

Solution to #4:

We see that $n = 4k + 2 = 2(2k + 1)$. However, 2 is even and $2k + 1$ is odd, which contradicts the condition that our two numbers must have the same parity. If we factor $2k + 1$, we can only do so as a product of two odd numbers, which doesn't help us.

If we choose 1 and $4k + 2$, we run into the same problem. Hence, no number of the form $4k + 2$, where k is a non-negative integer, can be expressed as a difference of two squares. ■

Solution to #5:

If a prime is greater than two, then it is odd. Hence, we see that $a = p$ and $b = q$ satisfy the conditions, which means any number of the form pq, where p and q are primes greater than two, can be expressed as a difference of two squares. ■

If $q = 2$ and $p > 2$, p is odd and q is even, which contradicts the condition that our two numbers must have the same parity. Hence, in that case, $\boxed{\text{the result does not hold}}$.

Solution to #6:

We can see that 675 is odd, which means the only way to factor 675 is as a product of odd numbers. Hence, if $ab = 675$, then a and b have the same parity, which means we don't have to worry about this condition.

We hence want to find the number of pairs (a, b) such that $ab = 675$ and $a < b$. Note that each pair (a, b) will give a different pair (x, y), and thus, a different way to express 675 as a difference of squares.

Since 675 is not a square number, it will have an even number of divisors, which can be grouped into pairs (a, b) such that $ab = 675$ and $a < b$. Hence, to find the number of such pairs, we want to find the number of divisors of 675 and divide that by 2.

To do this, we first factor 675 into its prime factorisation to obtain $675 = 3^3 \times 5^2$.
This means that each divisor will be of the form $3^s 5^t$, where $s \in \{0, 1, 2, 3\}$ and $s \in \{0, 1, 2\}$.

Since there are 4 possibilities for the value of 3^s and 3 for the value of 5^t, we have $4 \times 3 = 12$ different divisors.

To find the number of pairs of divisors, and hence the number of ways to express 675 as a difference of two squares, we divide by two to get $\frac{12}{2} = \boxed{6}$.