Tanish Dhir

# The Matrix

a)

1. $$\begin{pmatrix} 3 & -3 \\ 2 & 0 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & -1 & 5 \\ 0 & 3 & -2 \end{pmatrix} = \begin{bmatrix} 6 & -12 & 21 \\ 4 & -2 & 10 \\ 2 & 11 & -3 \end{bmatrix}$$

2. $$\begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 5 & -3 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 11 & -6 \\ 10 & -9 \end{pmatrix}$$

3. a) $$PQ = \begin{pmatrix} 2 & 3 & -1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} -2+0-5 \end{pmatrix} = \begin{pmatrix} -7 \end{pmatrix}$$

b) $$QP = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix} \begin{pmatrix} 2 & 3 & -1 \end{pmatrix} = \begin{bmatrix} -2 & -3 & 1 \\ 0 & 0 & 0 \\ 10 & 15 & -5 \end{bmatrix}$$

4. $$A = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 5 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix}$$

Dimensions of $A = 2 \times 3$ Dimensions of $B = 2 \times 2$

To multiply two matrices together, the number of columns of the first matrix has to be equal to the number of rows of the second matrix. Therefore, BA can be calculated but AB cannot.

$$BA = \begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 3 & -1 & 0 \\ -2 & 5 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 7 & 2 \\ 2 & -5 & -1 \end{pmatrix}$$

b)

1. Matrix multiplication is not commutative, so the order we use to multiply is important. In most cases, $AB \neq BA$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x & y \\ w & z \end{pmatrix} = \begin{pmatrix} (ax+bw) & (ay+bz) \\ (cx+dw) & (cy+dz) \end{pmatrix}$$

Not the same

$$\begin{pmatrix} x & y \\ w & z \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} (ax+cy) & (bx+dy) \\ (aw+cz) & (bw+dz) \end{pmatrix}$$

2. If $AB = 0$, we do not need to have A or B equal to $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. An example of this is when $A = \begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix}$ and $B = \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix}$, where a, b, c, d are real numbers.

3. $M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\quad M^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$$M^3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$M^4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Identity matrix

$$M^5 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = M$$

$\therefore M^{4n+1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\quad M^{2025} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\quad M^{2023} = M^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

4. $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad PX = XP$

One possible matrix that P could be is the identity matrix, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$IX = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \; XI = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

In addition to this, P could be any multiple of the identity matrix, e.g. $3I = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

identity matrix

$P = aI$

Scalar