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The Matrix

Matrix 3×**2** Matrix **2**×3 same, so they can be multiplied

1. $\begin{pmatrix} 3 & -3 \\ 2 & 0 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & -1 & 5 \\ 0 & 3 & 2 \end{pmatrix}$

$$\begin{pmatrix} 3\times2+(-3\times0) & 3\times(-1)+(-3\times3) & 3\times5+(-3\times2) \\ 2\times(2)+(0\times0) & 2\times(-1)+0\times3 & 2\times5+0\times2 \\ 1\times2+4\times0 & 1\times(-1)+4\times3 & 1\times5+4\times2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -12 & 2 \\ 4 & -2 & 10 \\ 2 & 11 & -3 \end{pmatrix}$$

2. $\begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 5 & -3 \\ -1 & 0 \end{pmatrix} =$

$$= \begin{pmatrix} 2\times5+(-1\times-1) & 2\times(-3)+(-1\times0) \\ 3\times5+5\times(-1) & 3\times(-3)+5\times0 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & -6 \\ 10 & -9 \end{pmatrix}$$

3. Let $P = (2\ 3\ -1)$ and let $Q = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}$
Find the products PQ and QP.

matrix 1×3 3×1

$PQ = (2\ 3\ -1) \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}$

$\boxed{\therefore PQ \neq QP}$

$= (2\times(-1)+3\times(0)+(-1\times5))$

$PQ = (-7)$

This is 1×1 matrix

$QP = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix} (2\ 3\ -1)$

this is a 3×3 matrix

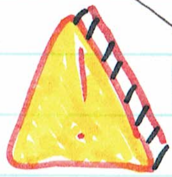
$$QP = \begin{pmatrix} (-1\times2) & (-1\times3) & (-1)\times(-1) \\ (0\times2) & (0\times3) & (0\times(-1)) \\ (5\times2) & (5\times3) & (5\times(-1)) \end{pmatrix} = \begin{pmatrix} -2 & -3 & 1 \\ 0 & 0 & 0 \\ 10 & 15 & -5 \end{pmatrix}$$

4. Let $A = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix}$

State which of the products AB and BA can be calculated and find the product.

$$AB = \underbrace{\overbrace{\begin{pmatrix} 3 & -1 & 0 \\ -2 & 5 & 1 \end{pmatrix}}^{3}}_{2} \overbrace{\begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix}}^{2} \, 2$$

Row × Column 2×3 Row × Column 2×2



NOT THE SAME.

∴ AB Cannot be calculated

- What about BA?

$$BA = \overbrace{\begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix}}^{2} \overbrace{\begin{pmatrix} 3 & -1 & 0 \\ -2 & 5 & 1 \end{pmatrix}}^{3} \, 2$$

They match! Let the working out begin!

2×2 $\begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix}$ 2×3 $\begin{pmatrix} 3 & -1 & 0 \\ -2 & 5 & 1 \end{pmatrix}$

$$\begin{pmatrix} 3 \times 3 + 2 \times (-2) & 3 \times (-1) + 2 \times 5 & 3 \times 0 + 2 \times 1 \\ 0 \times 3 + (-1) \times (-2) & 0 \times (-1) + (-1 \times 5) & 0 \times 0 + (-1 \times 1) \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 7 & 2 \\ 2 & -5 & -1 \end{pmatrix}$$

Part 2

1. No, $AB \neq BA$ for matrices.

For example $A = (1\,2\,3)$ and $B = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$AB = (1\,2\,3)\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$= (10)$

but $BA = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} (1\,2\,3)$

$= \begin{pmatrix} 3 & 6 & 9 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix}$

and (10) and $\begin{pmatrix} 3 & 6 & 9 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix}$ are very different.

2. No. For example, $A = \begin{pmatrix} 2 & 0 \\ 5 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 7 \\ 0 & 6 \end{pmatrix}$. $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

A is not $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, B is not $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ but $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Part 2

3. $M^1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$M^2 = M \times M$ i.e. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$M^3 = M \times M \times M$
$= M^2 \times M$ $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$M^4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ After 4, cycle pattern repeats

$M^5 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

$$\begin{array}{r} 505 \\ 4\overline{)2023} \\ \underline{20} \\ 23 \\ \underline{20} \\ 3 \end{array}$$

Remainder = 3 when divisor is 4

$\therefore M^{2023} = M^3$
$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

4. P is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$