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A Powerful Matrix

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Q^2 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$Q^3 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

$$Q^4 = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

$$Q^5 = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ 5 & 3 \end{pmatrix}$$

Fibonacci Sequence: 0, 1, 1, 2, 3, 5, 8, ...

Each matrix contains 3 consecutive terms of the Fibonacci sequence. When the exponent of Q increases by 1, the elements in the matrix increase to the next term in the sequence.

This happens because: $a = b+c, \therefore a+b = 2b+c$

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{bmatrix} (a+b) & a \\ (b+c) & b \end{bmatrix} = \begin{bmatrix} (2b+c) & (b+c) \\ (b+c) & b \end{bmatrix}$$

- Elements on the leading diagonal are the same.
- The top-left element is always the largest when the exponent > 1.
- The bottom-right element is always the smallest.