Repetitiously

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Solving example 1

Method 1.1 (Multiplication):

$$y = 0.25252525252525.....$$
$$y = 0.\dot{2}\dot{5}$$
$$100y = 25.\dot{2}\dot{5}$$
$$100y - y = 25.\dot{2}\dot{5} - 0.\dot{2}\dot{5}$$
$$99y = 25$$
$$\boxed{y = \frac{25}{99}}$$

Method 1.2 (Addition):

$$y = 0.25252525252525.....$$
$$y = 0.\dot{2}\dot{5}$$
$$100y = 25.\dot{2}\dot{5}$$
$$100y = 25 + 0.\dot{2}\dot{5}$$
$$100y = 25 + y$$
$$99y = 25$$
$$\boxed{y = \frac{25}{99}}$$

Method 2 (Sum of Geometric progression):

$$y = 0.25252525252525.....$$

$$\begin{aligned} y = &\ 0.2500000000000... \\ &\ 0.002500000000000.... \\ &\ 0.0000250000000.... \\ &\ 0.0000002500000.... \\ &\ 0.0000000025000.... \end{aligned}$$

$$y = \frac{25}{100} + \frac{25}{10000} + \frac{25}{1000000} + \ldots.$$

$$y = S_\infty = \frac{a_1}{1 - r}$$

$$r = \frac{\frac{25}{10000}}{\frac{25}{100}} = \frac{1}{100}, \quad r < 1(\text{converging})$$

$$y = \frac{\frac{25}{100}}{1 - \frac{1}{100}}$$

$$y = \frac{\frac{25}{100}}{\frac{99}{100}} = \frac{25}{99}$$

Solving example 2

Method 1.1 (Multiplication):

$$
\begin{aligned}
y &= 0.405405405405....\\
y &= 0.\dot{4}0\dot{5}\\
1000y &= 405.\dot{4}0\dot{5}\\
1000y - y &= 405.\dot{4}0\dot{5} - 0.\dot{4}0\dot{5}\\
999y &= 405
\end{aligned}
$$

$$\boxed{y = \frac{405}{999}}$$

Method 1.2 (Addition):

$$
\begin{aligned}
y &= 0.405405405405....\\
y &= 0.\dot{4}0\dot{5}\\
1000y &= 405.\dot{4}0\dot{5}\\
1000y &= 405 + 0.\dot{4}0\dot{5}\\
1000y &= 405 + y\\
999y &= 405
\end{aligned}
$$

$$\boxed{y = \frac{405}{999}}$$

Method 2 (Sum of Geometric progression):

$$y = 0.405405405405405...$$

$$\begin{aligned} y = {} & 0.405000000000000.... \\ & 0.000405000000000.... \\ & 0.000000405000000.... \\ & 0.000000000405000.... \\ & 0.000000000000405.... \end{aligned}$$

$$y = \frac{405}{1000} + \frac{405}{1000000} + \frac{405}{1000000000} + ..$$

$$y = S_\infty = \frac{a_1}{1 - r}$$

$$r = \frac{\frac{405}{1000000}}{\frac{405}{1000}} = \frac{1}{1000}$$

$$y = \frac{\frac{405}{1000}}{1 - \frac{1}{1000}}$$

$$y = \frac{\frac{405}{1000}}{\frac{999}{1000}} = \frac{405}{999}$$

Solving example 3

$$y = 0.8333333....$$

$$y = 0.8\dot{3}$$

$$10y = 8.\dot{3}$$

$$100y = 83.\dot{3}$$

$$100y - 10y = 83.\dot{3} - 8.\dot{3}$$

$$90y = 75$$

$$\boxed{y = \frac{75}{90}}$$

Solving example 4

$$y = 0.00277777777....$$

$$y = 0.002\dot{7}$$

$$1000y = 2.\dot{7}$$

$$10000y = 27.\dot{7}$$

$$10000y - 1000y = 27.\dot{7} - 2.\dot{7}$$

$$9000y = 25$$

$$\boxed{y = \frac{25}{9000}}$$

Generalised:

We can define a general form for any recurring decimal. For E.g.

54.003962796279627… can be rewritten as $54.003\dot{9}62\dot{7}$. It can further de divided into colours for easier interpretation, like this: $54.003\dot{9}62\dot{7}$. Where Blue numbers represent the number before the decimal point. The Red numbers represent non-recurring numbers and the Orange numbers represent the recurring numbers. This can be generalised as below:

$$\underbrace{a_1a_2a_3a_k}_{k}.\underbrace{b_1b_2b_3b_l}_{l}\underbrace{\dot{c}_1c_2c_3\dot{c}_m}_{m}$$

Let,

$$y = \underbrace{a_1a_2a_3a_k}_{k}.\underbrace{b_1b_2b_3b_l}_{l}\underbrace{\dot{c}_1c_2c_3\dot{c}_m}_{m}$$

$$10^l \cdot y = \underbrace{a_1a_2a_3a_k}_{k}\underbrace{b_1b_2b_3b_l}_{l}.\underbrace{\dot{c}_1c_2c_3\dot{c}_m}_{m}$$

$$10^{l+m} \cdot y = \underbrace{a_1a_2a_3a_k}_{k}\underbrace{b_1b_2b_3b_l}_{l}\underbrace{c_1c_2c_3c_m}_{m}.\underbrace{\dot{c}_1c_2c_3\dot{c}_m}_{m}$$

$$10^{l+m} \cdot y - 10^l \cdot y = \underbrace{a_1a_2a_3a_k}_{k}\underbrace{b_1b_2b_3b_l}_{l}\underbrace{c_1c_2c_3c_m}_{m}.\underbrace{\dot{c}_1c_2c_3\dot{c}_m}_{m} - \underbrace{a_1a_2a_3a_k}_{k}\underbrace{b_1b_2b_3b_l}_{l}.\underbrace{\dot{c}_1c_2c_3\dot{c}_m}_{m}$$

$$y(10^{l+m} - 10^l) = \underbrace{a_1a_2a_3a_k}_{k}\underbrace{b_1b_2b_3b_l}_{l}\underbrace{c_1c_2c_3c_m}_{m} - \underbrace{a_1a_2a_3a_k}_{k}\underbrace{b_1b_2b_3b_l}_{l}$$

$$y = \frac{a_1a_2a_3a_kb_1b_2b_3b_lc_1c_2c_3c_m - a_1a_2a_3a_kb_1b_2b_3b_l}{(10^{l+m} - 10^l)}$$

Notice how the recurring decimal (y) is converted into a fraction. No matter what the recurring decimal is, it can always be expressed in a form like above and can be simplified. Since, all the values of a_i, b_i, c_i are integers, any operation amongst them is an integer. Hence we can convert ant recurring decimal into a fraction

Example of the generalised form:

To test the method, we can use 54.003962796279627… as the example only.

$$y = \underbrace{54}_{2}.\underbrace{003}_{3}\underbrace{\dot{9}62\dot{7}}_{4}$$

$$10^3 \cdot y = \underbrace{54}_{2}\underbrace{003}_{3}.\underbrace{\dot{9}62\dot{7}}_{4}$$

$$10^{3+4} \cdot y = \underbrace{54}_{2}\underbrace{003}_{3}\underbrace{9627}_{4}.\underbrace{\dot{9}62\dot{7}}_{4}$$

$$10^{3+4} \cdot y - 10^3 \cdot y = \underbrace{54}_{2}\underbrace{003}_{3}\underbrace{9627}_{4}.\underbrace{\dot{9}62\dot{7}}_{4} - \underbrace{54}_{2}\underbrace{003}_{3}.\underbrace{\dot{9}62\dot{7}}_{4}$$

$$10^7 \cdot y - 10^3 \cdot y = \underbrace{54}_{2}\underbrace{003}_{3}\underbrace{9627}_{4} - \underbrace{54}_{2}\underbrace{003}_{3}$$

$$y(10^7 - 10^3) = 540039627 - 54003$$

$$y = \frac{540039627 - 54003}{(10^7 - 10^3)}$$

$$y = \frac{539985624}{(10^7 - 10^3)}$$

$$y = \frac{539985624}{9999000}$$

$$y = \frac{539985624}{9999000}$$

$$54.003962796279627\ldots = \frac{539985624}{9999000}$$