

TAKE THREE FROM FIVE

All ~~numbers~~ ^{Integers} can be of the form of either $3k-1$, $3k$, or, $3k+1$

In a set of five integers, there will be at least one form of number that will repeat itself more than once. [Pigeonhole pr.]

The numbers can be written as $\textcircled{-1}$, $\textcircled{0}$, $\textcircled{1}$ depending on the remainder it leaves after dividing by 3.

For example, 4 is $\textcircled{1}$, 11 is $\textcircled{-1}$ and 306 is $\textcircled{0}$

The possible combinations will always yield the sum to be divisible by 3

Type A: $\textcircled{1} + \textcircled{1} + \textcircled{1}$ divisible by 3

$\textcircled{-1} \textcircled{-1} \textcircled{-1}$ divisible by 3

$\textcircled{0} + \textcircled{0} + \textcircled{0}$ divisible by 3

Type B: $\textcircled{-1} + \textcircled{0} + \textcircled{1}$ is also div. by 3

★ Note: $\textcircled{-1}$ is $3k+2$ which is also $\equiv 2 \equiv -1 \pmod{3}$

This is just an alternate form of representing modular arithmetic. \mathbb{Z}

Below is a table of list of all possible combinations of (9), (0), (1)

No. of (-1)	No. of (0)	No. of (1)	Selection
0	0	5	A
0	4	1	A
0	3	2	A
0	2	3	A
0	1	4	A
0	5	0	A
1	0	4	A
1	1	3	A, B
1	2	2	B
1	3	1	A, B
1	4	0	A
2	0	3	A
2	1	2	B
2	2	1	B
2	3	0	A
3	0	2	A
3	1	1	A, B
3	2	0	A
4	0	1	A
4	1	0	A
5	0	0	A

selection type A means 3 of the same type can be chosen whereas B selection type means one from each group.